IIT JAM MATHEMATICS

Q1 The number of binary operations on a set A of four elements is

(A) 4^{16} (B) 16^4

(C) 16 (D) 4^4

Q2 The integral $\int_0^{4a}\int_{x^2/4a}^{2\sqrt{(ax)}} dxdy$ represents the area of the region enclosed by

(A) The parabola $y^2 = 4ax$ and the lines $y=0,\ x=4a$

(B) The parabola $y^2 = 4ay$ and the lines $x=0,\ y=4a$

(C) The parabolas $y^2 = 4ax$ and $x^2 = 4ay$

(D) The lines $x=0,\ x=4a,\ y=0,\ y=4a$

Q3 The value of $I = \int_0^8\int_{y^{1/3}}^2 \sqrt{x^4+1}dxdy$ is

(A) $\frac{3}{18}\left(17^{3/2}+1\right)$ (B) $\frac{3}{4}\left(17^{3/2}+1\right)$

(C) $\frac{3}{18}\left(17^{3/2}-1\right)$ (D) $\frac{3}{4}\left(17^{3/2}-1\right)$

Q4 Let A, B be $n\times n$ real matrices, which of the following statement is correct:

(A) $\text{Rank}(A+B) = \text{Rank } A + \text{Rank } B$

(B) $\text{Rank}(A+B) \le \text{Rank } A + \text{Rank } B$

(C) $\text{Rank}(A+B) = \min(\text{Rank } A, \text{Rank } B)$

(D) $\text{Rank}(A+B) = \max(\text{Rank } A, \text{Rank } B)$

Q5 let A, B be $n\times n$ matrices such that $BA + B^2 = I - BA^2$, where I is the n × n identity matrix. Which of the following is always true?

(A) A is non-singular

(B) B is non-singular

(C) $A+B$ non-singular

(D) AB is non-singular

Q6 Let J denote the matrix of order $n\times n$ with all entries 1 and let B be a $(3n)\times(3n)$ matrix given by

$$B = \begin{pmatrix} 0 & 0 & J \\ 0 & J & 0 \\ J & 0 & 0 \end{pmatrix}$$

Then the rank of B is

(A) $2n-1$ (B) $2n$

(C) 3 (D) 1

Q7 Consider the following system of equations:

$x_1 + x_2 - x_3 = 9$

$2x_1 + 3x_3 = a$

$3x_1 + x_2 + 2x_3 = 18$

what should a be so that this system is consistent?

(A) 10 (B) 1

(C) 2 (D) 9

Q8 Let A be a 3×4 matrix and B be a 4×3 matrix with real entries such that A B is non-singular. Consider the following statements:

P: Nullity of is $\mathbf{0}$.

Q: BA is a non-singular matrix.

Then

(A) Both P and Q are TRUE

(B) P is TRUE and Q is FALSE

(C) P is FALSE and Q is TRUE

(D) Both P and Q are FALSE

Q9 Let $f:\ \to$ be defined by, $f(x) = x^3 + x + a$, where $a \in$. Then:

(A) $f(x)$ has no real root for any a

(B) $f(x)$ has one real root if $a \ge 0$ and three real roots if $a < 0$

(C) $f(x)$ has exactly one real root for all a

(D) $f(x)$ has three real roots for all values of a

Q10 Let $f(x) = x^{1+\sin x}$, then $\lim\limits_{x\to\infty} x^{1+\sin(x)}$

(A) Does not exist

(B) Exist and equal to zero

(C) Exist infinitely

(D) Exist but non-zero and finite

Q11 Let $A = \begin{bmatrix} 1 & 0 & 2 \\ 1 & -2 & 0 \\ 0 & 0 & -3 \end{bmatrix}$ and I be the 3×3 identity matrix. If $6A^{-1} = aA^2 + bA + cI$ for $a,\ b,\ c \in$ then (a, b, c) equals

(A) (1,2,1) (B) $(1,-1,2)$

(C) (4,1,1) (D) (1,4,1)

Q12 Consider a sequence $\{x_n\}$ defined as $x_n = \frac{[\alpha]+[2\alpha]+\ldots+[n\alpha]}{n^2}$, where [.] is the greater integer function, then

(A) $\lim_{n\to\infty} x_n$ may or may not exist depending on α

(B) $\lim_{n\to\infty} x_n = \alpha$

(C) $\lim_{n\to\infty} x_n = \frac{\alpha}{2}$

(D) $\lim_{n\to\infty} x_n = 0$

Q13 Let $<a_n>$ be a sequence of real numbers defined as $a_{n+1} = \frac{a_n}{1+a_n^2}; a_1 \in \left(0, \infty\right)$ then

(A) $<a_n>$ is monotonic but not bounded

(B) $<a_n>$ is bounded but not monotonic

(C) $<a_n>$ is increasing and bounded

(D) $<a_n>$ is decreasing and bounded

Q14 Define

$f\left(x\right) = \lim_{n\to\infty}\left[\lim_{t\to 0}\frac{\sin^2(n!\pi x)}{\sin^2(n!\pi x)+t^2}\right]; x \in$ then

(A) $f\left(x\right)$ is continuous only on $x \in$

(B) $f\left(x\right)$ is continuous only on $x \in$ ^{c}

(C) $f\left(x\right)$ is continuous for all $x \in$

(D) $f(x)$ is nowhere continuous on $\mathbb{R}$

Q15 Consider sequences $<x_n>, <y_n>$, where x_n is arithmetic mean of $1, 2, \ldots, n$ and y_n is harmonic mean of $1, 2, \ldots, n$. Define $< a_n >, < b_n >$ as $a_n = \frac{1}{nx_n}$ and $b_n = \frac{y_n}{n}$, then

(A) $\sum a_n$ is converges but $\sum b_n$ is not convergent

(B) $\sum a_n$ is not convergent but $\sum b_n$ is convergent

(C) Both $\sum a_n$ and $\sum b_n$ converges

(D) Both $\sum a_n$ and $\sum b_n$ diverges

Q16 Consider the following

P: Let $f : [0.2] \to$ defines a continuous function such that $f(0) = f(2)$, then there exist $x \in (0, 1)$ such that $f(x) = f(x+1)$

Q: Let $f : S \subseteq \to$ defines a continuous function, then $f^{-1}\left(B\right)$ is closed for any closed subset $B \subseteq$

Then

(A) Both P & Q are true

(B) Only P is true

(C) Both P & Q are false

(D) Only P is false

Q17 The solution of the difference equation $y_{k+2} + y_k = 0$ is

(A) $y_k = A\cos\left(\frac{\pi k}{2}\right) + B\sin\left(\frac{\pi k}{2}\right)$

(B) $y_k = A\cos(k) + B\sin(k)$

(C) $y_k = A\cos(\pi k) + B\sin(\pi k)$

(D) $y_k = A\cos\left(\frac{\pi k}{3}\right) + B\sin\left(\frac{\pi k}{3}\right)$

Q18 Let G be group, $H \leq G$ such that $o(H)$ is odd and $o\left(\frac{G}{H}\right) = 2$ then

(A) H contain all the elements of even order

(B) H contain all the elements of odd order

(C) any odd prime p such that $p \mid o\left(G\right)$

(D) None of these

Q19 Let T be a linear operator on $V_3\left(\ \right)$ defined by

$T\left(a, b, c\right) = \left(3a, a-b, 2a+b+c\right)\ \forall a, b, c \in$

Then T^{-1} is given by

(A) $T^{-1}\left(a, b, c\right) = \left(\frac{a}{3}, \frac{a}{3} - b, c - a + b\right)$

(B) $T^{-1}\left(a, b, c\right) = \left(\frac{a}{3}, \frac{a-3b}{3}, c + b + a\right)$

(C) $T^{-1}\left(a, b, c\right) = \left(\frac{a}{3}, a - b, 2a + b + c\right)$

(D) $T^{-1}\left(a, b, c\right) = \left(\frac{a}{3}, \frac{a+3y}{3}, c + b - a\right)$

Q20 Let $f : {}^2 \to$ be such that $f\left(x, y\right) = x^3y + e^{xy^2}$. Then select the correct.

(A) $xf_x\left(x, y\right) - yf_y\left(x, y\right) = 0$

(B) $f_{xy} - f_{yx} = 0$

(C) $f_{xx}\left(x, y\right) - f_{yy}\left(x, y\right) = 0$

(D) $yf_x\left(x, y\right) - xf_y\left(x, y\right) = 0$

Q21 Let $f\left(x, y\right) = 2x^4 - 3x^2y + y^2$, then

(A) $f(x, y)$ does not has $(0, 0)$ as critical point

(B) $f(x, y)$ has maximum at $(0, 0)$

(C) $f(x, y)$ has neither maxima nor minima at (0,0)

(D) $f(x, y)$ has minimum at (0,0)

Q22 Let V be a finite dimensional vector space over $\mathbb{F}$, and $S \subseteq V$. Then choose the correct.

(A) If for every $x \in V - S$; $S \cup \{x\}$ is linearly dependent then S must be basis

(B) If every proper subset of S in linearly independent then S must be linearly independent.

(C) If S spans V and for every $x \in V - S$, $S \cup \{x\}$ is linearly dependent, then S is basis of V.

(D) None of the above

Q23 Let S be the family of orthogonal trajectories of the family of curves $2x^2 + y^2 = k$, for $k \in \mathbb{R}$ and $k > 0$. If $C \in S$ and C passes through the point $(1, 2)$, then C Also passes through

(A) $(4, -\sqrt{2})$
(B) $(2, -4)$
(C) $(2, 2\sqrt{2})$
(D) $(4, 2\sqrt{2})$

Q24 Let $T : \ ^3 \rightarrow \ ^3$ be a linear transformation satisfying.
$T(1,0,0) = (0,1,1),\ T(1,1,0) = (1,0,1)$
and $T(1,1,1) = (1,1,2)$ then
(A) $\text{Trace}(T) = 2$
(B) $\det(T) = -1$
(C) T is one - one but not onto
(D) T is neither one - one nor onto

Q25 Consider $\left(2x - 4y^2\right)\frac{dy}{dx} + y = 0$ such that $y(0) = 1$
Then which of the following is true?
(A) $y^2(2) = 1 + \sqrt{2}$ (B) $y^2(2) = 1 - \sqrt{2}$
(C) $y^2(2) = -1 + \sqrt{2}$ (D) $y^2(2) = -1 - \sqrt{2}$

Q26 Consider,
$f(x, y) = 2x^3 + 6xy^2 - 3y^3 - 150x$. Let $(a,\ b)$ be point of local minima. Then $a + b = 0$?
(A) 5 (B) 3
(C) 4 (D) 1

Q27 Which of the following is true about the given differential equation
$\left(1 - x^2\right)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + 4y = 0$?
(A) x=5 is not an ordinary point
(B) x=5 is an ordinary point
(C) x=4 is a singular point
(D) x=4is a irregular singular point

Q28 Let $f : [0, 2] \rightarrow$ be a function which is continuous on $[0, 2]$ and is differentiable on $(0, 2)$ with $f(0) = 1$.
Let $F(x) = \int_0^{x^2} f(\sqrt{t})dt$ for $x \in [0, 2]$.
If $F'(x) = f'(x)$ for all $x \in (0, 2)$, then $F(2)$ equals

(A) $e^2 - 1$ (B) $e^4 - 1$
(C) $e - 1$ (D) e^4

Q29 If a, b, and c are positive integers such that $a + b + c \leq 8$, the number of possible values of the ordered triplet ($a,\ b,\ c$) is
(A) 84 (B) 56
(C) 83 (D) 96

Q30 The straight lines l_1, l_2 and l_3 are parallel and lie in the same plane. A total numbers of m points are taken on l_1, n points on l_2, k points on l_3. The maximum number of triangles formed with vertices at these points is
(A) ${}^{(m+n+k)}C_3$
(B) ${}^{(m+n+k)}C_3 - {}^{m}C_3 - {}^{n}C_3 - {}^{k}C_3$
(C) ${}^{m}C_3 + {}^{n}C_3 + {}^{k}C_3$
(D) ${}^{m}C_3 \times {}^{m}C_3 \times {}^{k}C_3$

Q31 Let $*$ be defined, such that $a * b = \frac{ab}{2}$ then $\left(\ ^+, *\right)$ is
(A) Monoid
(B) Group
(C) Commutative Group
(D) All the above

Q32 Let A and B be $n \times n$ matrices with entries in a field F such that A.B=B.A=0 and A+B is invertible. Then which of the following is/are true?
(A) Rank (A) =Rank (B)
(B) Rank (A) +Rank (B) = n
(C) Nullity(A) + Nullity(B) = n
(D) A-B is invertible

Q33 If $M = \begin{bmatrix} 2 & 0 & 3 & 2 & 0 & -2 \\ 0 & 1 & 0 & -1 & 3 & 4 \\ 0 & 0 & 1 & 0 & 4 & 4 \\ 1 & 1 & 1 & 0 & 1 & 1 \end{bmatrix}$,
$b_1 = \begin{bmatrix} 5 \\ 1 \\ 1 \\ 4 \end{bmatrix}$ and $b_2 = \begin{bmatrix} 5 \\ 1 \\ 3 \\ 3 \end{bmatrix}$.
Then which of the following are true ?
(A) Both systems $MX = b_1$ and $MX = b_2$ are inconsistent
(B) Both systems $MX = b_1$ and $MX = b_2$ are consistent
(C) The system is $MX = b_1 - b_2$ consistent
(D) The systems is $MX = b_1 - b_2$ inconsistent

Q34 Pick out the case in which limit does not exists

(A) $\lim_{x\to 0}\frac{[x]}{x}$ where [.] is the greatest integer function.

(B) $\lim_{x\to 0}\left[\frac{1}{x}\right]$ where [.] is the greatest integer function.

(C) $\lim_{x\to 0}\frac{\cos\left(\frac{\pi}{2}\cos x\right)}{\sin(\sin x)}$

(D) $\lim_{x\to 0} x\sin\frac{1}{x}$

Q35 Consider the function $f(x)=\begin{cases}\frac{\sin 1/q}{\sin 1/p}, & x=p/q\\ 1+\sin(x-1), & x\in {}^{c}\cup\{0\}\end{cases}$

Let $S=\left\{c\in \ : \lim_{x\to c} f\left(x\right) \text{ exists}\right\}$. Then

(A) $|S|\geq 1$

(B) S is finite

(C) S is countably infinite

(D) S is empty

Q36 Let V be a vector space over $\mathbb{F}$ with dimension n. Let $T:V\to V$ be a linear transformation with only 1 as eigenvalue. Then which of the following must be true?

(A) $T-I=0$ (B) $(T-I)^{n-1}=0$

(C) $(T-I)^{n}=0$ (D) $(T-I)^{2n}=0$

Q37 Let $f(\ ,+)\to(\ ^{*},.)$ be a group homomorphism satisfying $\int_0^1 f\left(nx\right)dx=0; n\in\ .$

Then which of the following is/are true?

(A) There exists a unique function

(B) There does not exist such function

(C) There exist finite but more than one function

(D) There exist infinitely many functions

Q38 Let $f:[0,\infty)\to\ $ be defined as $f\left(x\right)=\begin{cases}\{x\}; & \text{if } [x] \text{ is even}\\ 1-\{x\}; & \text{if } [x] \text{ is odd}\end{cases}$ where $\{x\}$ is fractional part of function and [.] is the greatest integer function. Then which of the following is/are true ?

(A) f is continuous everywhere on $[0,\infty)$

(B) f nowhere continuous on $[0,\infty)$

(C) f has finitely many points of discontinuity

(D) f is continuous only on finite points

Q39 Let $f(x,y):\ ^2\to\ $ such that.

(A) $f(x,y)=\frac{x+y}{1+x^2}$, then $f(x,\ y)$ everywhere continuous

(B) $f\left(x,y\right)=\begin{cases}\frac{x^2y}{1+x} & ;x\neq -1\\ y & ;x=-1\end{cases}$ then $f(x,\ y)$ is everywhere continuous

(C) $f\left(x,y\right)=\begin{cases}\frac{x^2y-xy^3}{x^2+y^2} & ;(x,y)\neq(0,0)\\ 0 & ;\text{ if }(x,y)=(0,0)\end{cases}$ then $f_{xy}(0,0)=f_{yx}(0,0)$

(D) $f(x,y)=(x-ay)^6+(x+ay)^6$, then $f_{yy}=a^2f_{xx}$

Q40 Consider the differential equation $y''+P(x)y'+Q(x)y=0$ ... (1) where $P(x)$ and $Q(x)$ are continuous on interval containing 0 . Then which of the following function can not be solution of (1).

(A) $y(x)=x^2$ (B) $y(x)=x^3$

(C) $y(x)=x^3+x^2$ (D) $y(x)=x^2+x^4$

Q41 Let A be a set with 8 elements. If the number of commutative binary operations that can be defined on A is n, then $\log_2 n=$

Q42 If the integral $\int_0^1\int_0^1\frac{dxdy}{\sqrt{(1-x^2)(1-y^2)}}=\pi^2p^2$, then p is equal to

Q43 The value of the double integral $\iint_D\frac{xy}{\sqrt{1-y^2}}dxdy$ over the first quadrant of the circle $x^2+y^2=1$ is

Q44 Area of the region enclosed by the curves $y=x^2-x-2$ and $y=0$ is

Q45 If $M=\begin{pmatrix}3&4&0&0&0\\2&5&0&0&0\\0&9&2&0&0\\0&5&0&6&7\\0&0&4&3&4\end{pmatrix}$ then $det(M)$ is

Q46 If $a,\ b,\ c$ are the roots of the equation $x^3-2x+5=0$, then what is the value of $(a-b)(a-c)+(b-c)(b-a)+(c-a)(c-b)$?

Q47 Let $V = \{p : p(x) = a_0 + a_1 x + a_2 x^2,\ a_0, a_1, a_2 \in \ \}$ be the vector space of all polynomials of degree at most 2 over the real field . Let $T : V \to V$ be the linear operator given by $T(p) = (p(0) - p(1)) + (p(0) + p(1))x + p(0)x^2$.

Then the sum of the eigenvalues of T is ……

Q48 If $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$ then $\text{Trace}(A^{50})$ is

Q49 Let $k \in$, if $\lim_{x \to 0^+} \left(\sin kx + \cos kx + x \right)^{\frac{4}{x}} = e^{12}$, then the value of k is

Q50 Let $a_n = \left(2^n + n2^n \sin^2 \frac{n}{2}\right)^{\frac{1}{2n - n \cos \frac{1}{n}}}$ then $\lim_{n \to \infty} a_n$ is

Q51 Consider $<a_n>$ be a sequence of real numbers defined as $a_n = \frac{n^5 \cdot n!}{5 \cdot 6 \cdot 7 \cdot \ldots \cdot (5+n)}$, then $\lim_{n \to \infty} a_n$

Q52 Let $f : \ \to \ $ be defined as $f(x) = \begin{cases} x^3 \sin \frac{1}{x} & x \in \ \\ x^2 & x \in \ ^c \end{cases}$ .Then number of points at which f is continuous is

Q53 Let S_6 denote the symmetric group of six symbol and $n \in \ - \{1\}$ such that for each $x \in S_6$ there exist $g \in S_6$ with $g^{-1} x^n g = x$ then smallest value of n is

Q54 The least n to ensure that S_n has an element of order 50 is

Q55 If $T : P_3(\ ) \to P_4(\ )$ is a linear transformation given by $T(p(x)) = 2p'(x) + \int_0^x p(t)\,dt$ and if transformation matrix is $T = [a_{ij}]$ then a_{22} is

Q56 Let G be a group of order 30 and N_G be the collection of normal subgroups of G, thus maixmum cardinality of N_G for any G is...

Q57 Number of inner automorphism from $\mathbf{Z}_2 \times \mathbf{Z}_8 \to \mathbf{Z}_2 \times \mathbf{Z}_8$ is

Q58 Let y be the solution of the IVP $y'' + 2y' - 3y = 3;\ y(0) = 4,\ y'(0) = -7$. Then $\lim_{x \to \infty} e^x y(x)$ is

Q59 If the series $\sum_{n=1}^{\infty} 5(y)^{1 + \frac{1}{2} + \frac{1}{3} + \ldots + \frac{1}{n}}$ is convergent for $0 < y < a$ and divergent if $y > a$, then what is the value of a ? (upto two decimal)

Q60 If the constant term in the binomial expansion of $\left(\sqrt{x} - \frac{k}{x^2}\right)^{10}$ is 405 , then $|k|$ equals:

Answer Key

Q1	A
Q2	C
Q3	C
Q4	B
Q5	B
Q6	C
Q7	D
Q8	D
Q9	C
Q10	A
Q11	D
Q12	C
Q13	D
Q14	D
Q15	A
Q16	C
Q17	A
Q18	B
Q19	A
Q20	B
Q21	C
Q22	D
Q23	C
Q24	D
Q25	A
Q26	A
Q27	B
Q28	B
Q29	B
Q30	B
Q31	A, B, C, D
Q32	B, C, D
Q33	A, C
Q34	A, B
Q35	A, B
Q36	C, D
Q37	B
Q38	A, C
Q39	A, D
Q40	A, B, C, D
Q41	108~108
Q42	0.4~0.6
Q43	0.12~0.18
Q44	4.3~4.6
Q45	42~42
Q46	6~6
Q47	1~1
Q48	3~3
Q49	2~2
Q50	2~2
Q51	24~24
Q52	1~1
Q53	7~7
Q54	27~27
Q55	0~0
Q56	8~8
Q57	1~1
Q58	2~2
Q59	0.36~0.37
Q60	3~3

Hints & Solutions

Note: scan the QR code to watch video solution

Q1 Text Solution:

If $|A|= n$ then total number of binary operations on A are n^{n^2}

Here, $|A|= 4$ thus total number of binary operations on A are $4^{4^2} = 4^{16}$

Q2 Text Solution:

Given $\int_{x=0}^{4a} \int_{y=x^2/4a}^{2\sqrt{(ax)}} dxdy$

$\Rightarrow y = \frac{x^2}{4a};\ y = 2\sqrt{ax}$

$x = a\ ;\ x = 4a$

$\Rightarrow x^2 = 4ay;\ y^2 = 4ax$

Q3 Text Solution:

Given $R : y^{1/3} \le x \le 2$

$0 \le y \le 8$

After change of order of integration the limits will be

$D : 0 \le y \le x^3$

$0 \le x \le 2$

$\therefore I = \int_0^8 \int_{y^{1/3}}^2 \sqrt{x^4+1}dxdy$

$= \int_0^2 \left(\int_0^{x^3} \sqrt{x^4+1}dy\right)dx$

$= \int_0^2 x^3 \sqrt{x^4+1}dx$

Put $x^4 + 1 = t$

$\Rightarrow 4x^3dx = dt$

$= \frac{1}{4}\int_1^{17} \sqrt{t}dt$

$= \frac{1}{4} \times \frac{3}{2}(t)^{3/2}\Big|_1^{17}$

$= \frac{3}{8}\left(17^{3/2} - 1\right)$

Q4 Text Solution:

Take $A = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix};\ \text{Rank}\left(A\right) = 2$

and $B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix};\ \text{Rank}\left(B\right) = 2$

Then $A + B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix};\text{Rank}\left(A+B\right) = 0$

Hence, option (A), (C) and (D) are incorrect.

Let $C = A + B$

$\Rightarrow (\text{Col}(C)) \subseteq (\text{Col}(A)) + (\text{Col}(B))$

where $\text{Col}(C)$ is column space of C

$\Rightarrow \rho(C) \le \rho(A) + \rho(B).$

Q5 Text Solution:

Exp: $BA + B^2 = I - BA^2$

$\Rightarrow BA + B^2 + BA^2 = I$

$\Rightarrow B\left(A + B + A^2\right) = I$

$\Rightarrow B$ is invertible

For options (a) and (d):

Take $A = O_{n\times n}$

where $O_{n\times n}$ is zero matrix of order n.

Therefore, options (a) and (d) are false.

For option (c): Take $A = -B$.

Q6 Text Solution:

Given $B = \begin{pmatrix} 0 & 0 & J \\ 0 & J & 0 \\ J & 0 & 0 \end{pmatrix}_{3n\times 3n}$

where $J = \left[a_{ij} = 1\right]_{n\times n}$

Consider $n = 1 \Rightarrow J = (1)_{1\times 1}$

$\Rightarrow B = \begin{pmatrix} 0 & 0 & (1) \\ 0 & (1) & 0 \\ (1) & 0 & 0 \end{pmatrix}$

$\Rightarrow \rho(B)= 3$

Hence, options (A),(B) and (D) are discarded.

Therefore option (C) is correct.

Also we can prove the option (C)

$B = \begin{bmatrix} 0 & 0 & J \\ 0 & J & 0 \\ J & 0 & 0 \end{bmatrix}$

After applying finitely many row operations, we get

$B \sim \begin{bmatrix} J & 0 & 0 \\ 0 & J & 0 \\ 0 & 0 & J \end{bmatrix}$

which is a block diagonal matrix.

Now, using the fact that the rank of a block diagonal matrix

is equal to the sum of rank of each individual diagonal block, we get

$\rho(B) = \rho(J) + \rho(J) + \rho(J) = 3 \cdot \rho(J) = 3.1 = 3$

Q7 Text Solution:

Consider the augmented matrix

$$[A|b]=\left[\begin{array}{ccc|c} 1 & 1 & -1 & 9 \\ 2 & 0 & 3 & a \\ 3 & 1 & 2 & 18 \end{array}\right]$$

$$\sim\left[\begin{array}{ccc|c} 1 & 1 & -1 & 9 \\ 0 & -2 & 5 & a-18 \\ 0 & 0 & 0 & -9-a+18 \end{array}\right]$$

$\rho[A]=\rho(A|b)$ if

$-9-a+18=0$ i.e. a = 9

Q8 Text Solution:

As AB is a non-singular 3×3, matrix so $\rho(AB)=3$.

Also $\rho(AB)\le \min\{\rho(A),\rho(B)\}$

$3\le \rho(A)$

Here A is 3×4 matrix, hence $\rho(A)\le 3$

$\Rightarrow \rho(A)=3$

Similarly $\rho(B)=3$

and nullity of A is 4-3=1,

and statement P is false.

and $\rho(BA)\le \min\{\rho(A),\rho(B)\}$

$\rho(BA)\le 3$

So BA is singular, and statement Q is false.

Hence option (d) is correct.

Q9 Text Solution:

Given $f:\ \ \to$

$$f(x)=x^3+x+a$$
$$\Rightarrow f'(x)=3x^2+1>0\forall x$$

$\Rightarrow f$ is an increasing function, hence it cuts the x-axis only once.

Thus, it has exactly one real root of for all .

So option (b) is correct answer.

Q10 Text Solution:

Let $a_n=\frac{\pi}{2}+2n\pi$ and $b_n=\frac{3\pi}{2}+2n\pi$

Then $a_n\to\infty,\ b_n\to\infty$

but $\lim\limits_{n\to\infty} f(a_n)=\lim\limits_{n\to\infty}\left(\frac{\pi}{2}+2n\pi\right)^2\to\infty$

and $\lim\limits_{n\to\infty} f(b_n)=\lim\limits_{n\to\infty}\left(\frac{3\pi}{2}+2n\pi\right)^{1-1}=1$

Hence, by sequential criteria limit does not exists.

Q11 Text Solution:

$$A=\left[\begin{array}{cc|c} 1 & 0 & 2 \\ 1 & -2 & 0 \\ \hline 0 & 0 & -3 \end{array}\right]$$

Eigenvalues of A are 1, $-2,-3$

$\Rightarrow C_A(x)=(x-1)(x+2)(x+3)$

$=(x-1)(x^2+5x+6)$

$=x^3+5x^2+6x-x^2-5x-6$

$C_A(x)=x^3+4x^2+x-6$

Using, Cayley's Hamilton theorem

$\Rightarrow A_3+4A^2+A-6I=0$

$\Rightarrow 6I=A^3+4A^2+A$

Apply A^{-1} on both sides, we get

$\Rightarrow 6A^{-1}=A^2+4A+I$

$\Rightarrow 6A^{-1}=aA^2+bA+cI$ for $a,b,c\in$

Compare with above equation (i), we get

a = 1, b = 4, c = 1

So, (a, b, c) = (1, 4, 1)

Q12 Text Solution:

Note that $k\alpha-1\le[k\alpha]\le k\alpha\ \ \forall k\in$.

$\Rightarrow \frac{\alpha-1+2\alpha-1+\ldots+n\alpha-1}{n^2}\le\frac{[\alpha]+[2\alpha]+\ldots+[n\alpha]}{n^2}$

$\le\frac{\alpha+2\alpha+\ldots+n\alpha}{n^2}$

$\Rightarrow \frac{\frac{\alpha n(n+1)}{2}-n}{n^2}\le x_n\le\frac{\alpha n(n+1)}{2n^2}$

Clearly, $\lim\limits_{n\to\infty}\frac{\alpha n(n+1)}{2n^2}=\frac{\alpha}{2}$

$=\lim\limits_{n\to\infty}\frac{\frac{\alpha n(n+1)}{2}-n}{n^2}$

So, by squeeze theorem

$\lim\limits_{n\to\infty} x_n=\frac{\alpha}{2}$

Q13 Text Solution:

Consider $\frac{a_{n+1}}{a_n}=\frac{\frac{a_n}{1+a_n^2}}{a_n}$

$=\frac{1}{1+a_n^2}\le 1\quad\left[\text{since } a_n^2\ge 0\right]$

$\Rightarrow a_{n+1}\le a_n;\forall n\in$

$\Rightarrow <a_n>$ is monotonically decreasing

Now, for bounds using PMI

For $n=1$

$a_2=\frac{a_1}{1+a_1^2}>0$

Let $a_k=\frac{a_{k-1}}{1+a_{k-1}^2}>0$

Then $a_{k+1}=\frac{a_k}{1+a_k^2}>0$

$\Rightarrow\quad a_n>0\quad\forall n\in$

Q14 Text Solution:

Case (i) : Let $x\in$ then by choosing n to be sufficiently

large, $n!x$ can be made an integer, then $\sin(n!\pi x)=0$.

$\Rightarrow f(x)=\lim\limits_{t\to 0}\frac{0}{0+t^2}=0$

Case (ii) let $x\in\ ^c$, then $0<\sin^2(n!\pi x)<1$

$$\Rightarrow f\left(x\right) = \lim_{n\to\infty}\left[\lim_{t\to 0}\frac{1}{1+\frac{t^2}{\sin^2(n!\pi x)}}\right] = 1$$

$$\Rightarrow f\left(x\right) = \begin{cases} 0 & x \in \\ 1 & x \in {}^c \end{cases}$$

Hence, f is nowhere continuous on .

Q15 Text Solution:

$$a_n = \frac{1}{n\left(\frac{1+2+\ldots+n}{n}\right)}$$

$$= \frac{1}{1+2+\ldots+n}$$

$$= \frac{2}{n(n+1)} \sim \frac{1}{n^2}$$

$\Rightarrow \sum a_n$ converges.

Now, HM $\le$ AM

$$\Rightarrow \frac{1}{n}\times\left(\frac{n}{1+\frac{1}{2}+\ldots+\frac{1}{n}}\right) \le \frac{1+2+\ldots n}{n}\frac{1}{n}$$

$$\Rightarrow b_n \le \frac{n(n+1)}{2n^2} = \frac{n+1}{2n} = d_n(\text{say})$$

Since $\sum d_n$ doesn't converge

$\Rightarrow \sum b_n$ is also divergent using comparison test.

Q16 Text Solution:

For P take $f(x) = x(x-1)(x-2)$

Then $f(0) = 0 = f(2)$ but $f(x) \neq f(x+1)$.

For Q take $f : {}^+ \to$ as $f(x) = \log x$

Hence, both statements are false.

Q17 Text Solution:

Given $y_{k+2} + y_k = 0$

This is a second-order linear homogeneous recurrence with constant coefficients.

Assume a trial solution $y_k = r^k$.

$\Rightarrow r^{k+2} + r^k = 0$

Divide by r^k (assuming $r \neq 0$):

$\Rightarrow r^2 + 1 = 0.$

$\Rightarrow r = \pm i$

Thus the general solution is a linear combination of i^k and $(-i)^k$.

$i^k = \cos\left(\frac{\pi k}{2}\right) + i\sin\left(\frac{\pi k}{2}\right)$

Hence the general real solution is:

$y_k = A\cos\left(\frac{\pi k}{2}\right) + B\sin\left(\frac{\pi k}{2}\right)$

Q18 Text Solution:

since H has index 2 in G, it is normal subgroup of G and $|G : H| = 2$.

(A) Given $|H|$ is odd, so by Lagrange's theorem, every element

of H must have order dividing $|H|$, hence every element in H has odd order.

So H actually contains no nontrivial elements of even order at all.

Hence, option (A) is incorrect.

(B) Given $|G : H| = 2$ If follows that for every a in G, we have $(aH)^2 = H$.

Let a be an element of G, of order $2n+1$, then $H = a^{2n+1}$,

$H = \left((aH)^2\right)^n \cdot aH = aH$

Thus (a) is in H.

Hence, option (B) is correct.

(C)

$\left|\frac{G}{H}\right| = 2$

$\Rightarrow |G| = 2|H|$

and $|H|$ is some odd number.

It may very well have odd prime divisors.

For example, take $|H| = 3$, then |G|=6 and 3 divides |G|.

So we cannot conclude that no odd prime divides |G|.

Hence, option (C) is incorrect.

Q19 Text Solution:

Exp: Let $T(a,b,c) = (3a, a-b, 2a+b+c) = (p,q,r)$

$\Rightarrow 3a = p, a-b = q, 2a+b+c = r$

$$a = \frac{p}{3}, b = \frac{p}{3} - q = \frac{p-3q}{3}, c = r - p + q$$

$$\Rightarrow T^{-1}(p,q,r) = \left(\frac{p}{3}, \frac{p}{3} - q, r - p + q\right)$$

Q20 Text Solution:

$f_x = 3x^2y + e^{xy^2}y^2,\ f_y = x^3 + 2e^{xy^2}yx$

$f_{xx} = 6xy + y^4e^{xy^2},\ f_{yy} = 2xe^{xy^2} + 4x^2y^2e^{xy^2}$

$f_{xy} = 3x^2 + 2ye^{xy2} + 2xy^3e^{xy^2} = f_{yx}$

$\Rightarrow f_{xy} - f_{yx} = 0$

Q21 Text Solution:

$f_x = 8x^3 - 6xy,\ f_y = -3x^2 + 2y$

$f_x = 0,\ f_y = 0$

$\Rightarrow x\left(8x^2 - 6y\right) = 0$

$\Rightarrow x = 0,\ 8x^2 = 6y \ldots(1)$

$-3x^2 + 2y = 0$

$\Rightarrow 2y = 3x^2 \ldots(2)$

$From\ (1)\ and\ (2)$

$8x^2 = 9x^2$

$\Rightarrow x = 0$

$\Rightarrow y = 0$

$\Rightarrow$ $(0,0)$ is critical point

$f_{xx} = 24x^2 - 6y = 0$ at $(0,0)$

$f_{xy} = -6x = 0,$ at $(0,0)$

$f_{yy} = 2\ at\ (0,0)$

$\Rightarrow rt - s^2 = 0$

$Test\ fails$

Also $f(0,0)=0$

$\therefore f(x,y)-f(0,0)=(x^2-y)(2x^2-y)>0$ for $y<0$ or

$x^2>y>0<0$ for $y>x^2>\frac{y}{2}>0$

Hence, neither maximum nor minimum.

Q22 Text Solution:

For option (a), (c) any L.D. set S, will discard the options..

For (a), (c) are false

(b) $S=\{(1,10),(0,1),(1,1)\}\subseteq\ ^2$ every proper subset of S is L.I but S is L.D.

Hence, option (B) is incorrect.

Q23 Text Solution:

Step1: $4x+2y\frac{dy}{dx}=0$

Step2: Replace $\frac{dy}{dx}$ by $-\frac{dx}{dy}$

$$4x+2y\left(-\frac{dx}{dy}\right)=0$$

$$4x-2y\frac{dx}{dy}=0$$

$$4x=2y\frac{dx}{dy}$$

$$2x=y\frac{dx}{dy}$$

$\frac{2}{y}dy=\frac{1}{x}dx$

Step3: Integrating both sides

$2\log y=\log x+\log c$

$\log y^2=\log xc$

$\Rightarrow y^2=x.c$

This passing through $(1,2)$

$\Rightarrow 4=c$

$\Rightarrow y^2=4x\Rightarrow y=2\sqrt{2}$ at $x=2$

so, option C is true.

Q24 Text Solution:

Exp: $(x,y,z)=a(1,0,0)+b(1,1,0)+c(1,1,1)$

$\Rightarrow (x,y,z)=(a+b+c,b+c,c)$

$\Rightarrow c=z, b+c=y$

$\Rightarrow b=y-z$

\& $a+b+c=x$

$\Rightarrow a+y-z+z=x$

$\Rightarrow a=x-y$

$\Rightarrow T(x,y,z)=(x-y)T(1,0,0)+(y-z)T(1,1,0)+zT(1,1,1)$

$=(x-y)(0,1,1)+(y-z)(1,0,1)+z(1,1,2)$

$\Rightarrow T(x,y,z)=$

$(y-z+z,x-y+z,x-y+y-z+2z)$

$\Rightarrow T(x,y,z)=(y,x-y+z,x+z)$

$$\Rightarrow [T]_\beta=\begin{bmatrix}0&1&0\\1&-1&1\\1&0&1\end{bmatrix}$$

$\det(T)=0,\ \text{Trace }(T)=0$

$\Rightarrow T$ is neither one-one nor onto

Q25 Text Solution:

Given $\frac{dy}{dx}=\frac{-y}{2x-4y^2}$

$\Rightarrow \frac{dx}{dy}=\frac{4y^2-2x}{y}=4y-\frac{2x}{y}$

$\Rightarrow \frac{dx}{dy}+\frac{2x}{y}=4y$

So, I.F. $=e^{\int\frac{2}{y}dy}=e^{2\log y}=y^2$

So, solution is given by

$x\cdot y^2=\int 4y\cdot y^2+c$

$xy^2=y^4+c$

$\Rightarrow x=y^2+\frac{c}{y^2}$

$$y\left(0\right)=1\Rightarrow 0=1+c$$

$\Rightarrow c=-1$

So, $x=y^2-\frac{1}{y^2}$

$\Rightarrow xy^2=y^4-1$

$\Rightarrow y^4-xy^2-1=0$

$\Rightarrow y^2=\frac{x\pm\sqrt{x^4+4}}{2}$

But $y\left(0\right)=1$ so,

$y^2=\frac{x+\sqrt{x^2+4}}{2}$

$\Rightarrow y^2\left(2\right)=\frac{2\pm\sqrt{8}}{2}=1+\sqrt{2}$

Q26 Text Solution:

$f_x = 6x^2 + 6y^2 - 150 = 0$

and $f_y = 3y\left[4x - 3y\right] = 0$

then critical points are

$\left(-5,0\right), \left(5,0\right), \left(-3,-4\right), \left(3,4\right)$

Now, $f_{xx} = 12x$

$f_{xy} = 12y$

$f_{yy} = 12x - 18y$

$Let\ H = f_{xx} f_{yy} - \left(f_{xy}\right)^2$

$H\left(-5,0\right) > 0$ and $f_{xx}|_{(-5,0)} < 0$

$H\left(5,0\right) > 0$ and $f_{xx}|_{(5,0)} > 0$

$H\left(3,4\right) < 0$ and $H\left(-3,-4\right) < 0$

$\therefore \left(5,0\right)$ is point of local minima.

$\Rightarrow a + b = 5$

Q27 Text Solution:

Since $\left(1 - x^2\right)$ at $x = 5, 4$ is not zero then x=5 and x=4 are both ordinary points.

Q28 Text Solution:

$F\left(x\right) = \int_0^{x^2} f\left(\sqrt{t}\right) dt$

$\therefore F'\left(x\right) = 2xf\left(x\right)$

Also, $F'\left(x\right) = f'\left(x\right)$

$\Rightarrow f'\left(x\right) = 2xf\left(x\right)$

$\Rightarrow \int \frac{f'(x)}{f(x)} dx = \int 2x dx$

$\Rightarrow \log f\left(x\right) = x^2 + c_1$

$\Rightarrow f\left(x\right) = ce^{x^2}$

Now, $f\left(0\right) = 1$

$\Rightarrow c = 1$

$\Rightarrow f(x) = e^{x^2}$

$\Rightarrow F\left(2\right) = \int_0^4 e^t dt$

$= e^t|_0^4$

$= e^4 - 1$

Q29 Text Solution:

Given,

$a + b + c \le 8$

$a \ge 1, b \ge 1, c \ge 1$

As they are positive integers

We know

$a_1 + b_1 + c_1 \ldots + r\ th\ term = {}^{n+r-1}c_{r-1}$

Now,

$a_1 + 1 + b_1 + 1 + c_1 + 1 \le 8$

$\Rightarrow a_1 + b_1 + c_1 \le 5$

Now the solutions are

$a_1 + b_1 + c_1 = 0 = {}^2c_2 = 1$

$a_1 + b_1 + c_1 = 1 = {}^3c_2 = 3$

$a_1 + b_1 + c_1 = 2 = {}^4c_2 = 6$

$a_1 + b_1 + c_1 = 3 = {}^5c_2 = 10$

$a_1 + b_1 + c_1 = 4 = {}^6c_2 = 15$

$a_1 + b_1 + c_1 = 5 = {}^7c_2 = 21$

By adding all the values

$1 + 3 + 6 + 10 + 15 + 21$

$= 56$

Q30 Text Solution:

Here, total number of points are $(m + n + k)$ which must give ${}^{(m+n+k)}C_3$ number of triangles.

But m points on line I_1 taking 3 points at a time gives ${}^mC_3$ combinations which produce no triangle.

Similarly, ${}^nC_3$ and ${}^kC_3$ number of triangles cannot be formed.

Therefore, the required number of triangles is

${}^{(m+n+k)}C_3 - {}^mC_3 - {}^nC_3 - {}^kC_3$

Hence, the correct option is (B).

Q31 Text Solution:

Closure: Given a binary operation on $^+$ as $a * b = \frac{ab}{2}$.

Since, $a, b \in\ ^+$

$\Rightarrow a * b = \frac{ab}{2} \in\ ^+$

Hence, $\left(^+, *\right)$ is closed.

Identity:

$a * e = a = e * a \forall a \in\ ^+$

$\Rightarrow \frac{ae}{2} = a = \frac{ea}{2}$

$\Rightarrow e = 2 \in\ ^+$

Associativity:

$\left(a * b\right) * c = \left(\frac{ab}{2} * c\right)$

$= \frac{\frac{abc}{2}}{2} = \frac{abc}{4}$

$a * \left(b * c\right) = \left(a * \frac{bc}{2}\right)$

$= \frac{\frac{abc}{2}}{2} = \frac{abc}{4}$

Hence, $(^+, *)$ is associative.

Inverse:

$a * b = e$

$\Rightarrow \frac{ab}{2} = 2$

$\Rightarrow b = \frac{4}{a}\ as\ a \in\ ^+$

$\Rightarrow b \in\ ^+$

Hence, $(\ ^+, *)$ is a group.

Commutative:

$a * b = \frac{ab}{2}$

$= \frac{ba}{2}$

$= b * a$

Q32 Text Solution:

Result:

Let $T : V(F) \to U(F)$ be a linear transformation, where V and U are finite-dimensional vector space. Then $\rho(T) + \eta(T) = \dim(V(F))$

Solution:

A. Let n=2, and take A= $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

and $B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ then A.B=B.A= 0

and A+B is invertible.

But ρ(A)=2 and ρ(B)=0

(B),(C). Given that A B=0

$\Rightarrow \forall x \in \text{Im}(B)$ we have $x \in$ker(A)

$\Rightarrow$Range(B)$\subseteq$Ker(A)

$\Rightarrow \rho(B) \leq \eta(A)$

$\Rightarrow \rho(B) \leq n - \rho(A)$

$\Rightarrow \rho(A) + \rho(B) \leq n$......(1)

Also we know that

$\rho(A + B) \leq \rho(A) + \rho(B)$

$\Rightarrow n \leq \rho(A) + \rho(B)$.....(2)

from (1) & (2), we get

$\rho(A) + \rho(B) = n$

By Rank - nullity theorem,

$\eta(A) + \eta(B) = n$

D. $(A - B)^2 = A^2 + B^2 = (A + B)^2$, since A.B=B.A= 0

$\Rightarrow$ A-B is also invertible.

Hence,Option (B) ,(C) and (D) are correct.

Q33 Text Solution:

$$\left[\begin{array}{cccccc:ccc} 2 & 0 & 3 & 2 & 0 & -2 & 5 & 5 & 0 \\ 0 & 1 & 0 & -1 & 3 & 4 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 4 & 4 & 1 & 3 & -2 \\ 1 & 1 & 1 & 0 & 1 & 1 & 4 & 3 & 1 \end{array}\right]$$

$R_1 \leftrightarrow R_4$

$$\sim \left[\begin{array}{cccccc:ccc} 1 & 1 & 1 & 0 & 1 & 1 & 4 & 3 & 1 \\ 0 & 1 & 0 & -1 & 3 & 4 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 4 & 4 & 1 & 3 & -2 \\ 2 & 0 & 3 & 2 & 0 & -2 & 5 & 5 & 0 \end{array}\right]$$

$R_4 \leftrightarrow R_4 - 2R_1$

$$\sim \left[\begin{array}{cccccc:ccc} 1 & 1 & 1 & 0 & 1 & 1 & 4 & 3 & 1 \\ 0 & 1 & 0 & -1 & 3 & 4 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 4 & 4 & 1 & 3 & -2 \\ 0 & -2 & 1 & 2 & -2 & -4 & -3 & -1 & -2 \end{array}\right]$$

$R_4 \leftrightarrow R_4 + 2R_2$

$$\left[\begin{array}{cccccc:ccc} 1 & 1 & 1 & 0 & 1 & 1 & 4 & 3 & 1 \\ 0 & 1 & 0 & -1 & 3 & 4 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 4 & 4 & 1 & 3 & -2 \\ 0 & 0 & 1 & 0 & 4 & 4 & -1 & 1 & -2 \end{array}\right]$$

$R_4 \leftrightarrow R_4 + 2R_1$

Thus we have

$Rank(M) = 3$

Rank$[M : b_1] = 4$

Rank$[M : b_2] = 4$

Rank$[M : b_1 - b_2] = 3$

So, system is constant for $b_1 - b_2$, but not for b_1 and b_2.

Q34 Text Solution:

(A) $\lim\limits_{x \to 0^+} \frac{[x]}{x} = \lim\limits_{h \to 0} \frac{[0+h]}{h} = 0$

$\lim\limits_{x \to 0^-} \frac{[x]}{x} = \lim\limits_{h \to 0} \frac{[0-h]}{h} = \lim\limits_{h \to 0} \frac{-1}{h} = -\infty$

Hence, limit does not exists.

(B) Take $a_n = \frac{1}{n} \to 0\ as\ n \to \infty$

but $\lim\limits_{n \to \infty} f(a_n) = \lim\limits_{n \to \infty} [n] = \infty$

Hence, limit does not exists.

(C)

$\lim\limits_{x \to 0} \frac{\cos\left(\frac{\pi}{2} \cos x\right)}{\sin(\sin x)} = \lim\limits_{x \to 0} \frac{+\sin\left(\frac{\pi}{2} \cos x\right) \frac{\pi}{2} \sin x}{\cos(\sin x) \cos x} = 0$

(D) $\lim\limits_{x \to 0} x \sin \frac{1}{x} = 0$ since $\sin\left(\frac{1}{x}\right)$ is bounded.

Q35 Text Solution:

Let $\alpha \in$

Consider $x_n \to \alpha$

For option (a):

If $x_n \in$

$\Rightarrow x_n = \frac{p_n}{q_n}$ if $\alpha \neq 0$

$\Rightarrow f(x_n) = \frac{\sin\frac{1}{q_n}}{\sin\frac{1}{p_n}} = \frac{p_n}{q_n}\frac{\frac{1}{p_n}}{\sin\frac{1}{p_n}}\frac{\sin\frac{1}{q_n}}{\frac{1}{q_n}}$

$= \frac{p_n}{q_n} \to \alpha$

Now if $x_n \in \ ^c$

$\Rightarrow f(x_n) = 1 + \sin(x_n - 1) \to 1 + \sin(\alpha - 1)$

Limit exist when $\alpha = 1 + \sin(\alpha - 1)$

$\Rightarrow \alpha - 1 = \sin(\alpha - 1)$

$x = \sin x \quad \text{iff } x = 0$

$\Rightarrow \alpha - 1 = 0$

$\Rightarrow \alpha = 1$

$\Rightarrow S = \{1\}.$

Hence, option (A) and (B) are correct.

Q36 Text Solution:

Since 1 is the only eigen value of T and V is of dimension n.

So, characteristic polynomial is $(x-1)^n$.

By Cayley-Hamilton theorem

$(T-I)^n = 0 \Rightarrow (T-I)^{2n} = 0$

Hence, options (C) and (D) are correct.

Consider $n = 3$ and T is an operator

such that $A = \left[T\right] = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$,

then $A - I = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$ and $\left(A - I\right) \neq 0, \left(A - I\right)^2 \neq 0.$

Thus $\left(T - I\right) \neq 0$ and $\left(T - I\right)^2 \neq 0.$

Hence, options (A) and (B) are incorrect.

Q37 Text Solution:

A homomorphism from $f(\ ,+) \to (\ ^*,.)$ satisfies

$f(x+y) = f(x)\, f(y)$

$\Rightarrow f\left(x\right) = 1$ or $f\left(x\right) = a^x, a > 0.$

In either case $\int_0^1 f\left(nx\right)dx = \int_0^1 \left(f\left(x\right)\right)^n dx \neq 0$

as $f\left(x\right) > 0 \forall x \in \ .$

$\Rightarrow$ There does not exist such function.

Q38 Text Solution:

Given $f\left(x\right) = \begin{cases} \{x\} & ; [x] \text{ is even} \\ 1 - \{x\} & ; [x] \text{ is odd} \end{cases}$

$< br/ > \Rightarrow f\left(x\right) = \begin{cases} < br/ > 2m - x & x \in [2m-1, 2m) \\ < br/ > x - 2m & ; x \in [2m, 2m+1) < br/ > \end{cases} < br/ >$

then

$\lim_{x \to 2m^+} f\left(x\right) = \lim_{x \to 2m^-} f\left(x\right) = 0 = f\left(2m\right)$

and

$\lim_{x \to 2m+1^-} f\left(x\right) = \lim_{x \to 2m+1^+} f\left(x\right) = 1 = f\left(2m+1\right)$

$\therefore f$ is continuous $\forall x \in$ and set of discontinuities of $f = \phi$

Q39 Text Solution:

(a) Consider the sequence $< x_n > \to \alpha, < y_n > \to \beta$

$f(x_n, y_n) = \frac{x_n + y_n}{1 + x_n^n} \to \frac{\alpha + \beta}{1 + \alpha^2}$

$\Rightarrow$ everywhere continuous.

(b) Consider the sequence $< x_n > = -1 + \frac{1}{n}, < y_n > = \alpha, \ (\alpha \neq 0$ is arbitrary)

$\therefore f(x_n, y_n) = \frac{\left(-1+\frac{1}{n}\right)^2 \alpha}{1 - 1 + \frac{1}{n}}$

$= \frac{\left(1 + \frac{1}{n^2} - \frac{2}{n}\right)\alpha}{\frac{1}{n}}$

$= n\left(\alpha + \frac{\alpha}{n^2} - \frac{2\alpha}{n}\right)$

$= n\alpha + \frac{1}{n^2} - 2\alpha$

$= n\alpha + \frac{1}{n^2} - 2\alpha \to \infty$ as $n \to \infty$

Limit does not exist as $x \to -1$.

(c) $f_{yy}\left(0,0\right) = -1, f_{yx} = 1$

(d) $\left(x - ay\right)^6 + \left(x + ay\right)^6$ satisfy wave equation.

Q40 Text Solution:

For the initial value problem

$y'' + P\left(x\right)y' + Q\left(x\right)y = 0, \quad y\left(0\right) = y_0, y'\left(0\right) = y_1,$

there exists a unique solution near $x = 0$ (because P, Q are continuous).

Then the trivial solution $y \equiv 0$ satisfies the ODE.

By uniqueness, this means:

The only solution of the ODE with $y\left(0\right) = 0, y'\left(0\right) = 0$ is $y \equiv 0$.

(A) $y\left(x\right) = x^2$

$y\left(0\right) = 0, \quad y'\left(x\right) = 2x \Rightarrow y'\left(0\right) = 0.$

Nontrivial (not identically 0), but has $y\left(0\right) = 0, y'\left(0\right) = 0.$

$\Rightarrow$ Cannot be a solution.

Similarly options (B), (C) and (D) Cannot be a solution.

Q41 Text Solution:

We know if cardinality of a set is then number of commutative binary operations are $k^{\frac{k(k+1)}{2}}$.

Given set A such that $|A| = 8$.

Number of commutative binary operations on $A = 8^{\frac{8\times 9}{2}} = 8^{36}$.

$\Rightarrow n = 8^{36}$

The value of $\log_2 n = \log_2 8^{36} = \log_2 2^{3\times 36} = 3 \times 36 = 108$

.

Q42 Text Solution:

Given $\int_0^1 \int_0^1 \frac{dxdy}{\sqrt{(1-x^2)(1-y^2)}} = \pi^2 p^2$

$\Rightarrow \int_0^1 \frac{dy}{\sqrt{1-y^2}} \int_0^1 \frac{dx}{\sqrt{1-x^2}} = \pi^2 p^2$

$\Rightarrow \int_0^1 \frac{1}{\sqrt{1-y^2}} dy \left[\sin^{-1} x\right]_0^1 = \pi^2 p^2$

$\Rightarrow \frac{\pi}{2} \int_0^1 \frac{1}{\sqrt{1-y^2}} dy = \pi^2 p^2$

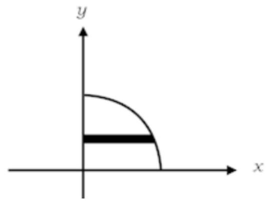
$\Rightarrow \frac{\pi}{2} \left[\sin^{-1} y\right]_0^1 = \pi^2 p^2$

$\Rightarrow \frac{\pi}{2} \times \frac{\pi}{2} = \pi^2 p^2$

$\Rightarrow p = 0.5$

Q43 Text Solution:

To find $\iint_D \frac{xy}{\sqrt{1-y^2}} dxdy$; where D is the 1^{st} quadrant of the circle $x^2 + y^2 = 1$.



In the first quadrant the limits are given by $0 \le y \le 1,\ 0 \le x \le \sqrt{1-y^2}$

$$\iint_D \frac{xy}{\sqrt{1-y^2}} dxdy$$

$$= \int_{y=0}^{1} \int_{x=0}^{\sqrt{1-y^2}} \frac{xy}{\sqrt{1-y^2}} dxdy$$

$$= \int_{y=0}^{1} \frac{y}{\sqrt{1-y^2}} \left[\frac{x^2}{2}\right]_0^{\sqrt{1-y^2}} dy$$

$$= \int_{y=0}^{1} \frac{y}{\sqrt{1-y^2}} \left[\frac{(1-y^2)}{2}\right]$$

$$= \frac{1}{2} \int_{y=0}^{1} y\sqrt{1-y^2}\, dy$$

Let $1 - y^2 = t$

$\Rightarrow \quad -2ydy = dt$

$\Rightarrow \quad \frac{1}{4} \int_0^1 t^{1/2} dt$

$\Rightarrow \quad \frac{1}{4} \frac{t^{3/2}}{3/2}\Big|_0^1$

$\Rightarrow \quad \frac{2 \times 1}{4 \times 3} = \frac{2}{12} = \frac{1}{6}$

Q44 Text Solution:

Area $= \iint_D dxdy$

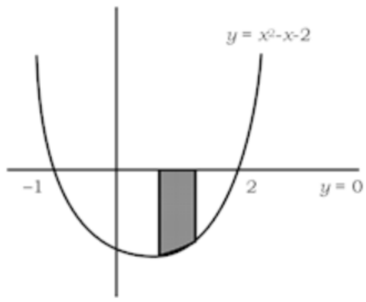
where D is the region enclosed by the curves

$y = x^2 - x - 2$ and $y = 0$

$\Rightarrow x^2 - x - 2 = 0$

$\Rightarrow x^2 - 2x + x - 2 = 0$

$\Rightarrow x = 2, -1$



Area

$= \int_{x=-1}^{2} \int_{y=x^2-x-2}^{0} dydx$

$= -\int_{x=-1}^{2} (x^2 - x - 2)dx$

$= -\left[\frac{x^3}{3} - \frac{x^2}{2} - 2x\right]_{-1}^{2}$

$= [(\frac{8}{3} - 2 - 4) - (-\frac{1}{3} - \frac{1}{2} + 2)]$

$= \frac{9}{2} = 4.5$

Q45 Text Solution:

Given matrix can be written as

$$M = \begin{bmatrix} \overset{A}{\begin{bmatrix} 3 & 4 & 0 \\ 2 & 5 & 0 \\ 0 & 9 & 2 \end{bmatrix}} & \begin{matrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{matrix} \\ \begin{matrix} 0 & 5 & 0 \\ 0 & 0 & 4 \end{matrix} & \underset{B}{\begin{bmatrix} 6 & 7 \\ 3 & 4 \end{bmatrix}} \end{bmatrix}$$

$\Rightarrow det(M) = det(A).det(B)$

$det(A) = \begin{vmatrix} 3 & 4 & 0 \\ 2 & 5 & 0 \\ 0 & 9 & 2 \end{vmatrix} = 14$

$det(B) = \begin{vmatrix} 6 & 7 \\ 3 & 4 \end{vmatrix} = 3$

$\Rightarrow det(M) = 42$

Q46 Text Solution:

Given equation a, b, c are the roots of the equation $x^3 - 2x + 5 = 0$.

$\Rightarrow a + b + c = 0 \ldots(1)$

$ab + bc + ca = -2 \ \ldots(2)$

$abc = -5 \ldots \ (3)$

$\Rightarrow (a-b)(a-c) + (b-c)(b-a) + (c-a)(c-b)$

$= a^2 - a(b+c) + bc + b^2 - b(a+c) + ac + c^2 - c(a+b) + ab$

$= 2a^2 + 2b^2 + 2c^2 + bc + ac + ab$

$= 2[(a+b+c)^2 - 2(ab+bc+ca)] + ab + bc + ca$

$= -3(ab+bc+ca)$

$= 6$

Q47 Text Solution:

Consider the standard basis $\{1, x, x^2\}$ of V.

$T(p) = (p(0) - p(1)) + (p(0) + p(1))x + p(0)x^2.$

$\Rightarrow T(1) = (1-1) + (1+1)x + 1 \cdot x^2 = 0 + 2x + x^2$

$T(x) = (0-1) + (0+1)x + 0 \cdot x^2 = -1 + x$

$T(x^2) = (0-1) + (0+1)x + 0 \cdot x^2 = -1 + x$

$[T] = \begin{pmatrix} 0 & -1 & -1 \\ 2 & 1 & 1 \\ 1 & 0 & 0 \end{pmatrix}$

The sum of the eigenvalues equals the **trace** of this matrix:

tr(T)=0+1+0=1.

So the sum of the eigenvalues of T is 1

Q48 Text Solution:

Given that, $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$

$\therefore A^2 = A \cdot A$

$= \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$

$\therefore A^4 = A^2 \cdot A^2$

$= \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix}$

$\therefore A^8 = A^4 \cdot A^4$

$= \begin{bmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 4 & 0 & 1 \end{bmatrix}$

$\therefore A^{16} = A^8 \cdot A^8$

$= \begin{bmatrix} 1 & 0 & 0 \\ 8 & 1 & 0 \\ 8 & 0 & 1 \end{bmatrix}$

$\Rightarrow A^{32} = \begin{bmatrix} 1 & 0 & 0 \\ 16 & 1 & 0 \\ 16 & 0 & 1 \end{bmatrix}$

$\therefore \quad A^{48} = A^{32} \cdot A^{16}$

$= \begin{bmatrix} 1 & 0 & 0 \\ 24 & 1 & 0 \\ 24 & 0 & 1 \end{bmatrix}$

$\therefore A^{50} = A^{48} \cdot A^2$

$= \begin{bmatrix} 1 & 0 & 0 \\ 25 & 1 & 0 \\ 25 & 0 & 1 \end{bmatrix}$

Hence required trace is 3.

Q49 Text Solution:

Given $\lim\limits_{x \to 0^+} \left(\sin kx + \cos kx + x \right)^{\frac{4}{x}} = e^{12}$

$\Rightarrow e^{\lim\limits_{x\to 0^+} \left(\sin kx + \cos kx + x - 1 \right) \times \frac{4}{x}} = e^{12}$

$\Rightarrow e^{\lim\limits_{x\to 0^+} 4\left[\frac{\sin kx}{x} + \frac{\cos kx - 1}{x} + 1 \right]} = e^{12}$

$\Rightarrow e^{\lim\limits_{x\to 0^+} 4\left| k \cos kx - k \sin kx + 1 \right|} = e^{12}$

$\Rightarrow e^{4 \cdot (k+1)} = e^{12}$

$\Rightarrow k + 1 = 3 \Rightarrow k = 2$

Q50 Text Solution:

Given $a_n = \left(2^n + n 2^n \sin^2 \frac{n}{2} \right)^{\frac{1}{2n - n \cos \frac{1}{n}}}$

$$\Rightarrow \quad \log a_n = \frac{1}{2n - n \cos \frac{1}{n}} \log \left(2^n \left(1 + n \sin^2 \frac{n}{2} \right) \right)$$

$$\Rightarrow \quad \log a_n = \frac{\log 2^n}{2n - n \cos \frac{1}{n}} + \frac{\log \left(1 + n \sin^2 \frac{n}{2} \right)}{2n - n \cos \frac{1}{n}}$$

Consider $b_n = \frac{n \log 2}{2n - n \cos \frac{1}{n}}$ and $c_n = \frac{\log \left(1 + n \sin^2 \frac{n}{2} \right)}{2n - n \cos \frac{1}{n}}$

$$\lim_{n\to\infty} b_n = \lim_{n\to\infty} \frac{\log 2}{2 - \cos \frac{1}{n}} = \log 2$$

$$\& \quad |c_n| \le \left| \frac{\log(1+n)}{n} \right| \to 0$$

$\Rightarrow \lim\limits_{n\to\infty} c_n = 0$ (By Squeeze theorem)

$$\Rightarrow \quad \lim_{n\to\infty} \log a_n = \lim_{n\to\infty} b_n + \lim_{n\to\infty} c_n = \log 2 + 0$$

$$\Rightarrow \quad \lim_{n\to\infty} a_n = 2$$

Q51 Text Solution:

$$a_n = \frac{n^5 \cdot n!}{5 \cdot 6 \cdot 7 \cdot \ldots \cdot (5+n)}$$

$$= \frac{1 \cdot 2 \cdot 3 \cdot 4 \cdot n^5 n!}{(5+n)!}$$

$$= \frac{1 \cdot 2 \cdot 3 \cdot 4 \cdot n^5 n!}{(n+5)(n+4)(n+3)(n+2)(n+1)n!}$$

$$= \frac{1 \cdot 2 \cdot 3 \cdot 4 \cdot n^5}{(n+5)(n+4)(n+3)(n+2)(n+1)}$$

$\Rightarrow \lim\limits_{n\to\infty} a_n = 4! = 24$

Q52 Text Solution:

f is continuous at $x = a$ if and only if $\forall <a_n>, <b_n> \to a$

$\Rightarrow \lim\limits_{n\to\infty} f(a_n) = \lim\limits_{n\to\infty} f(b_n) = f\left(a\right)$

Given $f\left(x\right) = \begin{cases} x^3 \sin\frac{1}{x} & x \in \\ x^2 & x \in {}^c \end{cases}$

If $<a_n> \to a; a_n \in$

$\& <b_n> \to a; b_n \in {}^c$

$\lim\limits_{n\to\infty} f(a_n) = a^3 \sin\frac{1}{a}$

and $\lim\limits_{n\to\infty} f(b_n) = a^2$.

$$\Rightarrow a^3 \sin\frac{1}{a} = a^2$$

$$\Rightarrow a^2\left[a\sin\frac{1}{a} - 1\right] = 0$$

$$\Rightarrow a = 0$$

Hence, f is continuous at exactly one point.

Q53 Text Solution:

< br/ > Exp: If $g^{-1}x^n g = x$

< br/ > < br/ >$\Rightarrow$ $o(x^n) = o(x)\forall x \in S_6$

< br/ >$\Rightarrow$ $(n, o(x)) = 1\forall x \in S_6$ < br/ >

and $o(x)$ can be $1, 2, 3, 4, 5, 6$.

Thus smallest n satisfying above is 7

Q54 Text Solution:

We know that $50 = 2 \times 25 = 2 \times 5^2$.

To obtain order 50, the cycle lengths must combine to give LCM 50.

Two natural cycles to generate this:

a 25-cycle (order 25)

a 2 -cycle (order 2)

Since $\mathrm{lcm}(25, 2) = 50$, this gives order 50 .

These cycles must be disjoint, so they use:

$25 + 2 = 27$ points .

This is the least n such that S_n contains an element of order 50 .

Q55 Text Solution:

$T(1) = x$

$T(x) = 2 + \frac{x^2}{2}$

$T\left(x^2\right) = 4x + \frac{x^3}{3}$

$T\left(x^3\right) = 6x^2 + \frac{x^4}{4}$

$$\Rightarrow T = \begin{bmatrix} 0 & 2 & 0 & 0 \\ 1 & 0 & 4 & 0 \\ 0 & \frac{1}{2} & 0 & 6 \\ 0 & 0 & \frac{1}{3} & 0 \\ 0 & 0 & 0 & \frac{1}{4} \end{bmatrix}$$

Hence, $a_{22} = 0$

Q56 Text Solution:

since there only 4 groups upto isomorphism of order 30 , namely

$\mathbf{Z}_{30}, A_4 \times \mathbf{Z}_5, S_3 \times \mathbf{Z}_5, \mathbf{Z}_3 \times D_5$

$\mathbf{Z}_{30}$ has 8 normal subgroups

$A_4 \times \mathbf{Z}_5$ has 6 normal subgroups

$S_3 \times \mathbf{Z}_5$ has 4 normal subgroups

$\mathbf{Z}_5 \times D_5$ has 6 normal subgroups

Q57 Text Solution:

$$\text{Exp: } G = \mathbf{Z}_2 \times \mathbf{Z}_8, o(G) = 16$$

$$o(Z(G)) = o(G) = 16 \quad \text{(since } G \text{ is abelian)}$$

$$\Rightarrow G = Z(G)$$

$$\frac{G}{Z(G)} \approx \mathrm{inn}(G)$$

$$\text{Since } o(\mathrm{inn}(G)) = \frac{o(G)}{o(Z(G))} = 1$$

< br/ >

Q58 Text Solution:

Solve the homogeneous equation

$y'' + 2y' - 3y = 0$

Auxiliary Equation is given by

$m^2 + 2m - 3 = 0$

$\Rightarrow m = 1, -3$

$\Rightarrow y_c(x) = C_1 e^x + C_2 e^{-3x}$

To find the particular integral

Let $y_p(x) = A$

$\Rightarrow 0 + 2 \times 0 - 3A = 3$

$\Rightarrow A = -1$

Hence the general solution is given by

$y(x) = C_1 e^x + C_2 e^{-3x} - 1$

$From\ y\left(0\right) = 4:$

$\Rightarrow C_1 + C_2 = 5 \ldots(1)$

$y'(0) = -7$

$\Rightarrow C_1 - 3C_2 = -7 \ldots(2)$

$From\ \left(1\right)\ and\ \left(2\right)$

$C_1 = 2,\ C_2 = 3.$

$\Rightarrow y(x) = 2e^x + 3e^{-3x} - 1$

$\Rightarrow \lim\limits_{x\to\infty} e^{-x} y = 2$

Q59 Text Solution:

Logarithmic test:

Let $\sum u_n$ be a series of positive real numbers and $\lim\limits_{n\to\infty} n \log\frac{u_n}{u_{n+1}} = l$.

Then $\sum u_n$ is

(a) convergent if $l > 1$

(b) divergent if $l < 1$

(c) test fails if $l = 1$

Now, let $u_n = 5y^{\left(1+\frac{1}{2}+\ldots+\frac{1}{n}\right)}$

$\Rightarrow \frac{u_n}{u_{n+1}} = y^{-\frac{1}{n+1}}$

$\Rightarrow \lim\limits_{n\to\infty} n\log\frac{u_n}{u_{n+1}} = \lim\limits_{n\to\infty} n\log y^{-\frac{1}{n+1}}$

$= \lim\limits_{n\to\infty} n\log\left(y^{-1}\right)^{\frac{1}{n+1}}$

$= \lim\limits_{n\to\infty} \frac{n}{n+1}\log\left(y^{-1}\right)$

$= \lim\limits_{n\to\infty} \frac{n}{n+1}\log\left(\frac{1}{y}\right)$

Now the series is convergent if

$\log\frac{1}{y} > 1 \Rightarrow \frac{1}{y} > e$

$\Rightarrow y < \frac{1}{e}$

The series is divergent if

$\log\frac{1}{y} < 1 \Rightarrow \frac{1}{y} < e \Rightarrow y > \frac{1}{e}$.

$\Rightarrow a = \frac{1}{e}$

Q60 Text Solution:

General term $= T_{r+1} = {}^{10}C_r(\sqrt{x})^{10-r}\cdot\left(-\frac{k}{x^2}\right)^r$

$= {}^{10}C_r(-k)^r\cdot x^{\frac{10-r}{2}-2r} = {}^{10}C_r(-k)^r\cdot x^{\frac{10-5r}{2}}$

For constant term, $\frac{10-5r}{2} = 0$ then $r = 2$

$\because {}^{10}C_2(-k)^2 = 405$

$\Rightarrow k^2 = \frac{405\times 2}{10\times 9} = \frac{81}{9} = 9$

Hence, $|k| = 3$