Real test

Physics

Q1 Express the complex number $\frac{i-1}{\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}}$ into polar form:
(A) $\sqrt{2}\left(\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}\right)$
(B) $\sqrt{2}\left(\cos\frac{5\pi}{12}+i\sin\frac{5\pi}{12}\right)$
(C) $2\left(\cos\frac{3\pi}{4}+i\sin\frac{3\pi}{4}\right)$
(D) $\sqrt{3}\left(\cos\frac{5\pi}{6}+i\sin\frac{5\pi}{6}\right)$

Q2 In an experiment, the activity of 1.2 milligrams of radioactive potassium chloride (chloride of isotope K-40) was found to be 170s^{-1}. Taking molar mass of to be 0.075 kg mole^{-1}. The half-life of K-40 is______$\times 10^{16}$ (In s) Avogadros number $= 6.0 \times 10^{23}\ mole^{-1}$
(A) 1.57 (B) 2.74
(C) 3.92 (D) 7.54

Q3 A particle of mass m is moving under a one dimensional potential $V(x) = -ax + bx^2$ where a>0, b>0. What is the frequency of oscillation about the stable equilibrium?
(A) $\sqrt{\frac{2b}{m}}$ (B) $\sqrt{\frac{m}{2b}}$
(C) $\sqrt{\frac{b}{m}}$ (D) $\sqrt{\frac{m}{b}}$

Q4 A particle in thermal equilibrium has only 3 possible states with energies $-\varepsilon,\ 0,\ and,\ \varepsilon$. If the system is maintained at a temperature $T >> \frac{\varepsilon}{k_B}$ then the average energy of the particle can be approximated to,
(A) $\frac{2\varepsilon^2}{2k_BT}$
(B) $-\frac{2\varepsilon^2}{3k_BT}$
(C) $\frac{\varepsilon^2}{k_BT}$
(D) 0

Q5 A diffraction grating has 6000 slits per cm and is illuminated with monochromatic light of wavelength 500 nm. The grating is used in third-order diffraction (m = 3). What is the minimum number of slits that must be illuminated to just resolve two spectral lines separated by $\Delta\lambda$ = 0.01nm?
(A) 15679 (B) 16734
(C) 16973 (D) 16667

Q6 In an FCC crystal, which condition on Miller indices (hkl) determines that an X-ray reflection is allowed?
(A) h,k,l are all even or all odd.
(B) h+k+l is even.
(C) At least one of h,k,l is even.
(D) Any integer triplet (h,k,l) gives a reflection.

Q7 An n-type semiconductor is doped with donor atoms of concentration N_D. If the temperature is increased to a very high value, which of the following statements is correct?
(A) Electron concentration remains approximately equal to N_D
(B) Hole concentration decreases with temperature
(C) The semiconductor behaves like an intrinsic semiconductor
(D) Fermi level moves closer to donor level

Q8 Two observers S and S' are in standard configuration: S' moves with constant velocity v along $+x$-axis relative to S. A particle moves in S with position $x(t) = 5t^2$ (in meters and seconds). Which statement is correct about its motion in S'?

(A) The acceleration of the particle in S′ is different from its acceleration in S.

(B) The acceleration of the particle in S′ is the same as in S.

(C) The acceleration becomes zero in S′ if v=10 m/s.

(D) The velocity of the particle in S′ remains the same as in S for all t.

Q9 Compute the following binary subtraction:

$11010101_2 \; - \; 10111011_2 = \; ?$

(A) 00011010 (B) 00011001

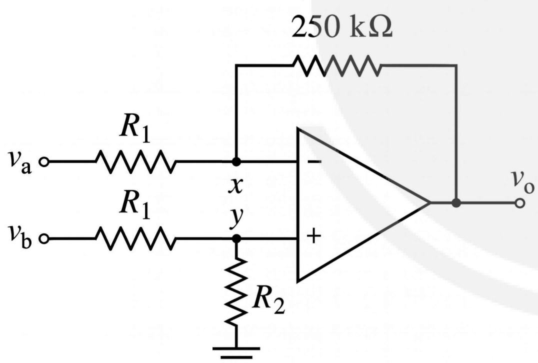
(C) 00111010 (D) 01011010

Q10 $\left(\frac{-1+\sqrt{-3}}{2}\right)^{100} + \left(\frac{-1-\sqrt{-3}}{2}\right)^{100}$ is equal to

(A) 2 (B) 0

(C) -1 (D) 1

Q11 For the op-amp shown below, find the value of R_1 *and* R_2 for the output $v_0 = -5v_a + 3v_b$



(A) $50\ k\Omega$ *and* $50\ k\Omega$

(B) $25\ \ k\Omega$ *and* $50\ k\Omega$

(C) $25\ \ k\Omega$ *and* $25\ k\Omega$

(D) $25\ \ k\Omega$ *and* $75\ k\Omega$

Q12 A photon of energy E = 1.25 MeV strikes a free electron at rest. After scattering, the electron is measured to have kinetic energy K = 0.40 MeV. Assuming mc^2 = 0.511 MeV for the electron, find the photon's scattering angle is φ and the electron's recoil angle is θ (both measured from the incident photon direction).

(A) $\varphi = 0^\circ, \theta = 90^\circ$

(B) $\varphi = 41.136^\circ, \theta = 36.653^\circ$

(C) $\varphi = 26.136^\circ, \theta = 81.653^\circ$

(D) $\varphi = 36.136^\circ, \theta = 41.653^\circ$

Q13 A satellite of mass $m = 1200$ kg moves in an elliptical orbit around Earth with semi-major axis a and eccentricity $e = 0.8$. Its energy is E and angular momentum is L. It is transferred to a circular orbit with radius equal to the apogee of the original orbit. Choose the correct statement:

(A) Total energy increases, angular momentum decreases

(B) Total energy increases, angular momentum increases

(C) Total energy decreases, angular momentum decreases

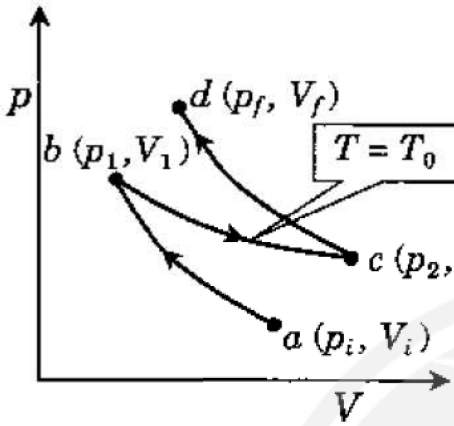
(D) Total energy decreases, angular momentum increases

Q14 A particle of mass m sits at the centre of a square of side $2a$. Four identical massless springs (each of spring constant k) connect the mass radially to the four vertices of the square; all springs are unstretched in equilibrium, so the mass is at the centre. The mass is displaced by a small vector $\mathbf{x}$ in the plane and released. Neglect any out-of-plane motion. What is the angular frequency ω of the resulting small oscillatory motion? (Answer is independent of the direction of x.)

(A) $\omega = \sqrt{\frac{2k}{m}}$ (B) $\omega = \sqrt{\frac{4k}{m}}$

(C) $\omega = \sqrt{\frac{2k}{3m}}$ (D) $\omega = \sqrt{\frac{k}{m}}$

Q15 One mole of an ideal monatomic gas in an initial state with pressure, p_i, and volume, V_i, is to be taken to a final state d with $p_f = B^2 p_i$ and $V_f = V_i / B$ through the path $a \to b \to c \to d$ as shown in the figure below for a particular value of $B (> 1)$. Here $c \to b$ and $c \to d$ are adiabatic while $b \to c$ is an isotherm with temperature T_0. States b and c correspond to (p_1, V_1) and (p_2, V_2), respectively.



Find the total work done by the gas in terms of $p_i,\ V_i,\ T_0$ *and* B

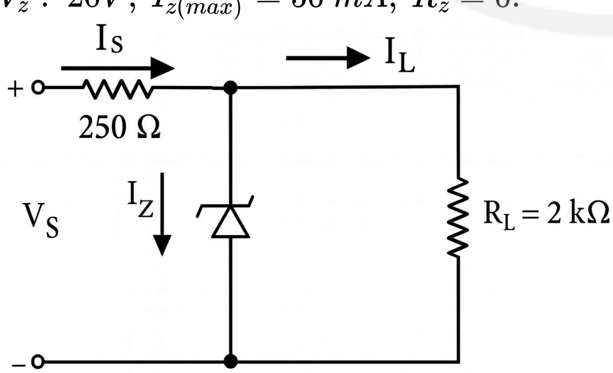
(A) $W = \frac{RT_0}{2}\ \ln\ B + (1 - B) p_i V_i$

(B) $W = \frac{RT_0}{2}\ \ln\ B + \frac{3}{2}(1 - B) p_i V_i$

(C) $W = \frac{RT_0}{2}\ \ln\ B + B p_i V_i$

(D) $W = \ln\ B + \frac{3}{2}(1 - B) p_i V_i$

Q16 for the zener regulated circuit given below, determine the range of input voltage (V_s) for the zener diode to remain in "ON" state. Given: $V_z\ :\ 20V,\ I_{z(max)} = 50\ mA,\ R_z = 0.$



(A) 12.5 V to 30 V

(B) 22.5 V to 35 V

(C) 15 V to 25 V

(D) 20.5 V to 38.5 V

Q17 If A and B are square matrices of size $n \times n$, then which of the following statement is not true?

(A) $\det(AB) = \det(A)\det(B)$

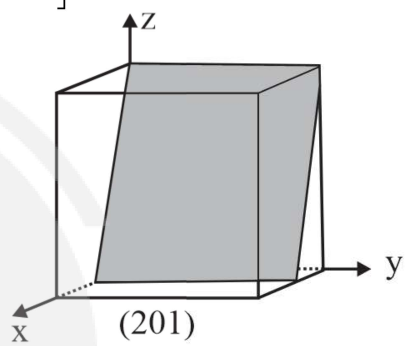
(B) $\det(kA) = k^n \det(A)$

(C) $\det(A + B) = \det(A) + \det(B)$

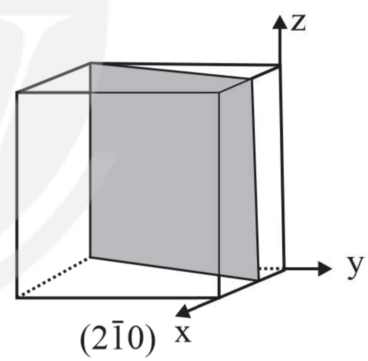
(D) $\det\left(A^T\right) = 1/\det\left(A^{-1}\right)$

Q18 Choose the right representation of miller indices $\left[2\ \bar{1}\ 0\right]$

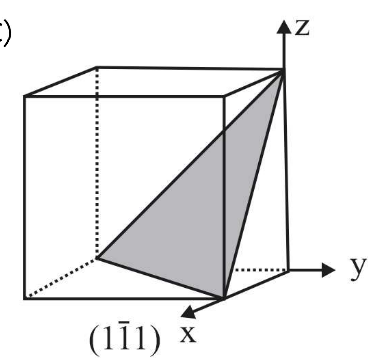
(A)



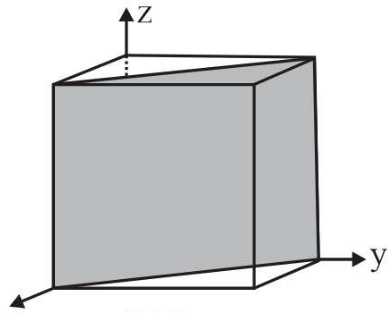
(B)



(C)



(D)



(110)

Q19 An inertial observer sees two events E_1 and E_2 happening at the same location but $6\mu s$ apart in time. Another observer moving with a constant velocity v (with respect to the first one) sees the same events to be $9\mu s$ apart. The spatial distance between the events, as measured by the second observer, is approximately

(A) $1434m$ (B) $3394m$

(C) $2000m$ (D) $5000m$

Q20 Which of the following 2×2 matrices is **both** Hermitian and unitary?

(A) $A = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

(B) $B = \begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix}$

(C) $C = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$

(D) $D = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$

Q21 In Gaussian units, the magnetic field in empty space is:

$\mathbf{B} = B_0 e^{ax} \sin(ky - \omega t)\, \hat{z}.$

What is the y-component of the electric field?

(A) $-\frac{ac}{\omega} B_0 \sin\left(ky - \omega t\right)$

(B) $-\frac{ac}{\omega} B_0 e^{ax} \cos\left(ky - \omega t\right)$

(C) $-B_0 \sin(ky - \omega t)$

(D) 0

Q22 A system of non-interacting Fermi particles with fermi energy E_n has the density of states proportional to $\sqrt{E}$, where E is the energy of a particle. The average energy particle at temperature T = 0 is?

(A) $\frac{1}{6}\ E_f$ (B) $\frac{1}{5}\ E_f$

(C) $\frac{2}{5}\ E_f$ (D) $\frac{3}{5}\ E_f$

Q23 A mixture of two isotopes, A and B, of the same element undergoes a solid-liquid phase transition with a distinct coexistence curve on a pressure-temperature (P-T) diagram. The Clausius-Clapeyron equation for each isotope, represented by subscripts A and B, is:

$\ln\left(\frac{P_L}{P_V}\right) = \frac{\Delta H_{vap}}{RT}$

where ΔH_{vap} is the molar enthalpy of vaporization, R is the gas constant, P_L is the pressure of the liquid phase, and P_V is the pressure of the vapor phase. Both isotopes have different molar masses and slightly different ΔH_{vap} values.

During a carefully controlled experiment, the mixture is slowly heated at constant pressure (isobaric) through the coexistence region. Surprisingly, instead of observing separate solidification points for each isotope, a single, **sharp** transition peak is detected at a specific temperature, T_c.

(A) The sharp peak results from a complex interplay of isotope fractionation during vaporization and the Clausius-Clapeyron relationship for each isotope.

(B) The observed peak reflects a critical point where the liquid and vapor phases become indistinguishable for both isotopes.

(C) The sharp transition signifies a eutectic point, where both isotopes solidify simultaneously at a minimum melting temperature.

(D) The single peak indicates the complete solid-liquid phase transition of the heavier isotope at T_c, followed by a continuous melting process for the lighter isotope.

Q24 A sphere of radius R has a non-uniform volume charge density:

$\rho(r) = kr$

where k is a constant and r is the radial distance from the center. The sphere is centered at the origin. The electric potential at the center of the sphere ($r = 0$) with respect to infinity is:

(A) $\frac{kR^3}{3\varepsilon_0}$

(B) $\frac{3kR^3}{\varepsilon_0}$

(C) $\frac{2kR^3}{3\varepsilon_0}$

(D) $\frac{kR^3}{\varepsilon_0}$

Q25 There is a small black dot at the centre C of a solid glass sphere of refractive index μ. When seen from outside, the dot will appear to be located:

(A) Away from the C for all values of μ

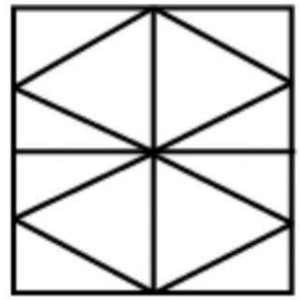
(B) At C for all values of μ.

(C) At C for μ=1.5, but away from C for μ not equal to 1.5

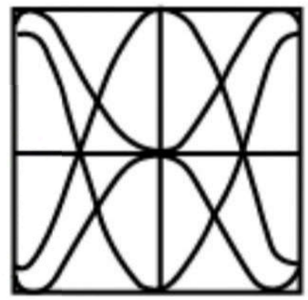
(D) At C for 2<μ<1.5

Q26 In a oscilloscope the input to the horizontal plates is a 100 Hz voltage signal. The lissajous pattern (A), (B) and (C) will be generated when different frequency voltage signals are applied to vertical plates. Match each lissajous pattern to the corresponding frequency by

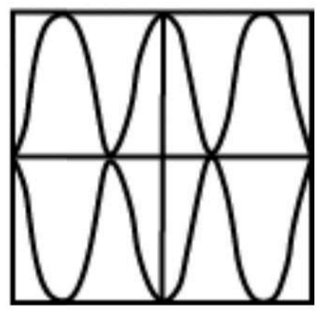
A.



B.



C.



P. $f_y = 50$ Hz

Q. $f_y = 66.66$ Hz

R. $f_y = 150$ Hz

S. $f_y = 150$ Hz

T. $f_y = 200$ Hz

U. $f_y = 300$ Hz

(A) $A - P; B - S; C - T$

(B) $A - Q; B - S; C - T$

(C) $A - P; B - S; C - U$

(D) $A - P; B - S; C - U$

Q27

Two rails of a railroad track are insulated from each other and from the ground, and are connected by a millivoltmeter. What is the reading of the millivoltmeter when a train travels at the speed of 90 km/hr down the track? Assume that the vertical component of the earth's magnetic field is 0.2 gauss and that the tracks are separated by 2 m. Use $1 \text{ gauss} = 10^{-4} \text{ T}$.

(A) 10 (B) 1
(C) 0.2 (D) 180

Q28 $\begin{pmatrix} x & y \end{pmatrix} \begin{pmatrix} 5 & -7 \\ 7 & 3 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = 15$

The matrix equation above represents
(A) circle of radius $\sqrt{15}$
(B) an ellipse of semi minor axis $\sqrt{5}$
(C) An ellipse of semi major axis 5
(D) A hyperbola

Q29 The radius of gyration *k* of a hollow sphere is formed by removing a smaller solid sphere of radius ***b*** and mass m from a larger solid sphere of radius ***a*** and mass *M*, assuming uniform density.

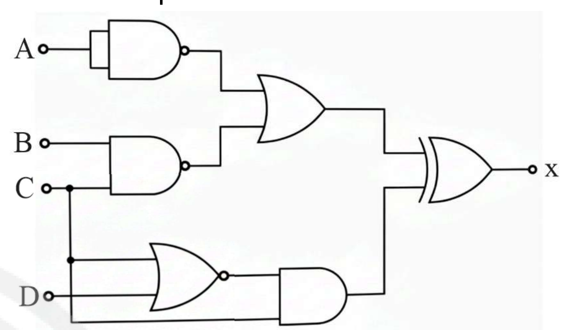
(A) $k = \sqrt{\frac{3}{5}\left(\frac{a^3-b^3}{a^5-b^5}\right)}$ (B) $k = \sqrt{\frac{5}{4}\left(\frac{a^3-b^3}{a^5-b^5}\right)}$
(C) $k = \sqrt{\frac{2}{5}\left(\frac{b^5-a^5}{b^3-a^3}\right)}$ (D) $k = \sqrt{\frac{2}{5}\left(\frac{a^5-b^5}{a^3-b^3}\right)}$

Q30 A particle is located in a three-dimensional cubic well width L with impenetrable walls. The sum of the energies of the third and the fourth levels and degeneracy of the fourth level is given by
(A) $10\pi^2\hbar^2/mL^2$, 3
(B) $10\pi^2\hbar^2/3mL^2$, 4
(C) $11\pi^2\hbar^2/mL^2$, 4
(D) $15\pi^2\hbar^2/mL^2$, 3

Q31 A periodic function $f(x) = x^2$ defined on $-\pi < x < \pi$ is expanded in a Fourier series. Which of the following statements are **correct**?

(A) The constant (average) term is $\frac{\pi^2}{3}$
(B) The cosine coefficients are $a_n = \frac{4(-1)^n}{n^2}$
(C) All sine coefficients b_n are zero.
(D) Only odd cosine harmonics appear in the expansion (i.e. terms $\cos(2k+1)x$

Q32 Solve the expression for x:



(A) $\overline{A.B.C}$ (B) $\overline{A.B}.C$
(C) $\overline{A} + \overline{B} + \overline{C}$ (D) $\overline{A} + \overline{B} + C$

Q33 A particle of mass mm with energy E=9 eV is incident from the left on a step potential:

$$V(x) = \begin{cases} 0 & x < 0 \\ 4 \text{ eV} & x > 0 \end{cases}$$

Given $\frac{\hbar^2}{2m} = 3.81 \text{ eV}.\text{Å}^2$ For this case $E > V_0$, calculate the reflection coefficient R and transmission coefficient T.

(A) 0.9576 (B) 0.0424
(C) 0.0213 (D) 0.9787

Q34 An object is initially kept at rest and breaks up into three fragments of mass ratio $1:2:3$. If the speed of the heaviest fragment just after explosion is v_0. It is given that the three fragments move in the same plane and make equal angles with each other. Then which of the following statement(s) is/are correct?
(A) Speed of the heaviest fragment is v_0.
(B) Speed of the lightest fragment is $3v_0$.
(C) The vector sum of momenta of three fragments must not be zero.

(D) All are correct.

Q35 For a smooth vector field $\vec{F}$ and an open surface S with boundary ∂S, which statements are true?

(A) Reversing the orientation of ∂S changes the sign of Stokes' theorem result.

(B) If $\nabla \times \vec{F} = \vec{0}$, then the line integral around **any** closed loop is zero.

(C) If the line integral around a closed loop is zero, then $\nabla \times \vec{F} = \vec{0}$ everywhere.

(D) Stokes' theorem is valid only for flat surfaces.

Q36 A point charge q is placed at the **center** of a grounded conducting spherical shell of radius R. The image-charge configuration that enforces V(R)=0 is:

(A) A single image charge −q at the center.

(B) An image charge of magnitude $-\frac{R}{d}q\ at\ r' = \frac{R^2}{d}$ placed on the same radial line outside the sphere .

(C) No finite image charge; equivalently, a single "image at infinity" that adds a uniform potential shift

(D) Two equal and opposite image charges placed symmetrically outside the sphere.

Q37 A system has 3 normal modes of vibration, with frequencies $\omega_1, \omega_2 = 2\omega_1$ and $\omega_3 = 3\omega_1$. If system having energy is less than the $\frac{11}{2}\hbar\omega$ at temperature T then which of the following is/are relavent statement/s

(A) First excited state is doubly degenrate.

(B) Partition function is
$Z = e^{-\beta\hbar\omega} + 2^{-\beta 4\hbar\omega} +^{-\beta 5\hbar\omega}$

(C) Second excited state is doubly degenrate.

(D) Partition function is
$Z = e^{-\beta 3\hbar\omega} + e^{-\beta 4\hbar\omega} + 2e^{-\beta 5\hbar\omega}$

Q38 The phase velocity of deep-water waves is given by, $v^2 = \frac{g\lambda}{2\pi} + \frac{2\pi\sigma}{\rho\lambda}$. Where, $g = 9.8\ \mathrm{ms}^{-2}, \rho = 1000\mathrm{kgm}^{-3}$ and $\sigma = 7.2 \times 10^{-2}\mathrm{Nm}$ (σ is the surface tension of water).Determine the value of group velocities and (λ_0) which is the wavelength of the waves which do not disperse in water.

(A) $v_g = 23\mathrm{cms}^{-1}$

(B) $v_g = 7\mathrm{cms}^{-1}$

(C) $\lambda_0 = 1.7\ cm$

(D) $\lambda_0 = 2.5\ cm$

Q39 Two handheld radio transceivers equipped with dipole antennas are positioned at a significant fixed distance apart. If the transmitting antenna is oriented vertically, what proportion of the maximum power will be received by the receiving antenna when it is tilted away from the vertical by a certain angle?

(A) Power will receive maximum 90 degrees.

(B) No power will receive at 90 degrees.

(C) Only 50% power will receive at 45 degrees.

(D) Only 75% power will receive at 30 degrees.

Q40 Three electrons are placed in the same p-orbital shell (l=1) of an atom. Which of the following statements are correct?

(A) All three electrons can have the same spin projection $m_s = +\frac{1}{2}$.

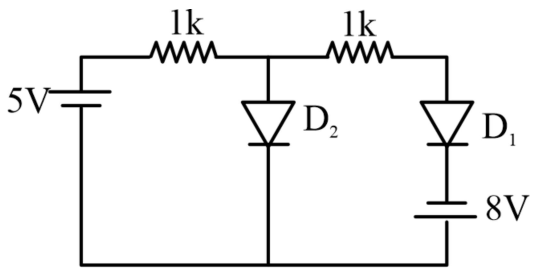
(B) The maximum number of electrons that can occupy a p-subshell is 6.

(C) The total spin S of the three electrons can be $S = \frac{3}{2}$.

(D) At least two electrons must have opposite spins in the same orbital.

Q41 A cubic crystal has lattice constant $a = 5.64\ \mathring{A}$. It is irradiated by Cu–Kα X-rays of wavelength $\lambda = 1.54\ \mathring{A}$. For the $(1\,1\,1)$ planes ($d_{111} = a/\sqrt{3}$), what is the largest possible order n of Bragg reflection (integer $n \geq 1$) that can occur from these planes?

Q42 Assuming that diodes are ideal the current in D_2 is



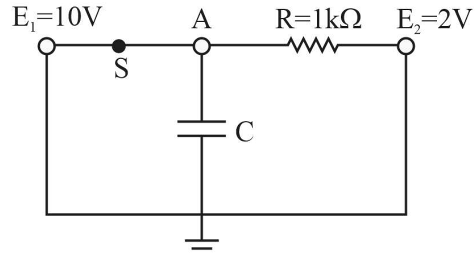
Q43 The blackbody at a temperature of 6000K emits a radiation whose intensity spectrum peaks at 600nm. If the temperature is reduced to 300K, the spectrum will peak at,

Q44 A group of five students is participating in a tug-of-war competition. The students have masses of 60kg, 70kg, 80kg, 90kg, and 100kg respectively. If they are pulling on opposite ends of a rope, calculate the point along the rope where the center of mass of the group lies.

Q45 A circle is given by the equation $2x^2 + 2y^2 + 8x - 20y + 10 = 0$. The area of a square whose side equals the radius of the circle is ____

Q46 A dipole consists of charges ±2.0 μC separated by 2a = 0.20 m along the x-axis. Find the approximate electric potential (in volts) in terms ....$\times 10^3 V$ at the point P(0.60 m,0.80 m) using the dipole approximation (r>>a). Take $k = 9 \times 10^9 N.m^2/C^2$.

Q47 A capacitor $C = 1.0\ \mu\mathrm{F}$ is initially fully charged to $E_1 = 10\ \mathrm{V}$ while switch S is open. At $t = 0$ the switch SS is closed, connecting the capacitor through a resistor $R = 1.0\ \mathrm{k\Omega}$ to a second DC source $E_2 = 2.0\ \mathrm{V}$. (The capacitor's other terminal is connected to the negative terminal common to both supplies.)



What is the voltage across the capacitor at $t = 2.4$ ms?

Q48 A 7 L vessel contains 3.50 moles of gas at a pressure of 1.6×10^6 Pa. The average kinetic energy of the gas molecules in the vessel is $x \times 10^{-20}$ Joule. The value of x is (Answer upto three place of decimal)

Q49 A block of unknown mass is attached to a spring with a spring constant of 6.50 N/m and undergoes simple harmonic motion with an amplitude of 10.0 cm. When the block is halfway between its equilibrium position and the end point, its speed is measured to be 30.0 cm/s.Then the mass of the block is n kg. The value of n is(Answer upto three place of decimal)

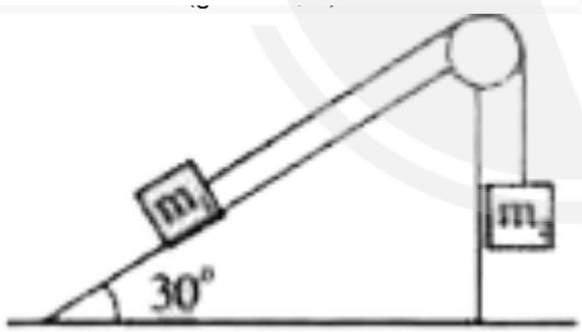
Q50 A particle is moving with $0.8c\hat{j}$ (c is the speed of light) in an inertial frame S_1. Frame S_2 is moving with velocity $0.8c\hat{i}$ with respect to S_1. Let E_1 and E_2 be the respective energies of the particle in the two frames. Then, $\frac{E_2}{E_1}$ is (round off to two decimal places).

Q51 A silicon semiconductor has donor concentration $N_D = 5 \times 10^{16}\ \mathrm{cm}^{-3}$ and acceptor concentration $N_A = 2 \times 10^{16}\ \mathrm{cm}^{-3}$ at T=300 K. The intrinsic carrier concentration is $n_i = 1.5 \times 10^{10}\ \mathrm{cm}^{-3}$. Assuming complete ionization of dopants, what is the Fermi level relative to the intrinsic Fermi level $E_F - E_i$ in eV? ($kT = 0.0259\ \mathrm{eV}$). (give naswer upto two decimal places.)

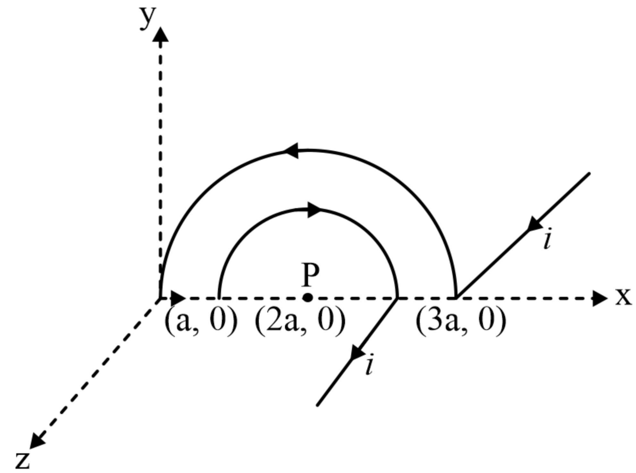
Q52 A quantum harmonic oscillator has a wave function $\psi\left(x\right) = Ae^{-\alpha x^2} H_n\left(\sqrt{\alpha}x\right)$, where (A) is the normalization constant,(α) is a positive real number,(H_n) is the Hermite polynomial of order (n), and (x) is the position. Given that the expectation value of the position is zero, then the expectation value of the momentum for the ground state n=0

Q53 Water flows through a horizontal pipe whose diameter decreases from $D_1 = 0.10\ \mathrm{m}$ to $D_2 = 0.05\ \mathrm{m}$. The pressure at the wider section is $P_1 = 2.0 \times 10^5\ \mathrm{Pa}$ and the velocity is $v_1 = 1.5\ \mathrm{m/s}$. Assuming incompressible, non-viscous flow, calculate the pressure P_2 at the narrow section on units of ....$\times 10^5 Pa.$
Take water density $\rho = 1000\ \mathrm{kg/m^3}$

Q54 A block of mass m_1= 4 kg on a smooth inclined plane of 30° is connected by a cord over a small, frictionless pulley to a second block of mass m_2= 5 kg hanging vertically. Calculate the acceleration with which the block moves and also the tension in the cord. Take g = 10 m/sec^2.



Q55 In the figure given below, the magnetic field at the point P is $B = x \times \frac{\mu_0 I}{\pi a}\sqrt{1+\pi^2}$. The value of x is ____ (answer upto one decimal place).



Q56 A system is composed of nitrogen N_2(224gm) and oxygen O_2(56gm). The ratio of rms speed of N_2 and O_2 at 20°C will be (Answer upto one decimal place)

Q57 An optical line of wavelength 5000$\mathring{A}$ in the spectrum of light from a star is found to be red-shifted by 2$\mathring{A}$. Let v be the velocity at which the star is receding. Ignoring relativistic effects, what is the value of $\frac{c}{v}$?

Q58 A stationary ${}^{12}_{6}\mathrm{C}$ nucleus is bombarded by a neutron ${}^{1}_{0}\mathrm{n}$ and the following nuclear reaction occurs:

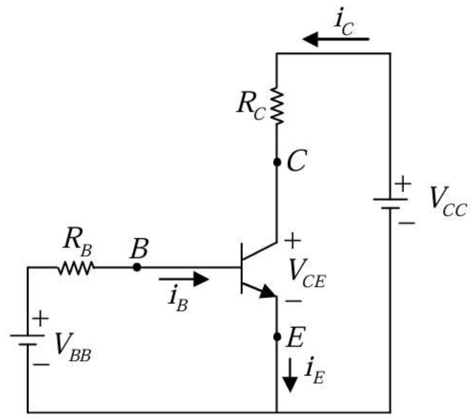
$${}^{1}_{0}\mathrm{n} + {}^{12}_{6}\mathrm{C} \longrightarrow {}^{13}_{6}\mathrm{C} + \gamma$$

Masses (in u) are:

$$m_{\mathrm{n}} = 1.008, \quad m_{\mathrm{C12}} = 12.000, \quad m_{\mathrm{C13}} = 13.003$$

If the kinetic energy of the incident neutron is 5.0 MeV, calculate the kinetic energy of the ${}^{13}_{6}\mathrm{C}$ nucleus after the reaction.
(Take $1\ \mathrm{u} = 931.494\ \mathrm{MeV}/c^2$ and neglect photon recoil; give answer in MeV, rounded to one decimal place.)

Q59 Consider the transistor circuit shown. Assume $V_{BEQ} = 0.7V,\ V_{BB} = 6V$ and leakage is negligible. What value of R_B (in kilo-ohms) is required if the base current is to be I_B= 4 μA?



Q60 The finite region R is defined by the inequalities $2 \leq x^2 + y^2 \leq 4$ and $1 \leq x^2 - y^2 \leq 2$. Given further that x>0 and y>0, the value of integral $\iint_R x^3 y^3 dx.\, dy$ is_____.(upto 2 decimal places)

Answer Key

Q1	(B)
Q2	(C)
Q3	(A)
Q4	(B)
Q5	(D)
Q6	(A)
Q7	(C)
Q8	(B)
Q9	(A)
Q10	(C)
Q11	(A)
Q12	(D)
Q13	(B)
Q14	(A)
Q15	(B)
Q16	(B)
Q17	(C)
Q18	(B)
Q19	(C)
Q20	(A)
Q21	(B)
Q22	(D)
Q23	(A)
Q24	(A)
Q25	(B)
Q26	(C)
Q27	(B)
Q28	(B)
Q29	(D)
Q30	(A)
Q31	(A, B, C)
Q32	(A, C)
Q33	(C, D)
Q34	(A, B)
Q35	(A, B)
Q36	(B, C)
Q37	(C, D)
Q38	(A, C)
Q39	(B, C, D)
Q40	(B, C, D)
Q41	4~4
Q42	5~5
Q43	11.5~12.5
Q44	2.2~2.3
Q45	24~24
Q46	2.01~2.36
Q47	2.5~3.5
Q48	0.795~0.798
Q49	0.542~0.543
Q50	1.45~1.85
Q51	0.34~0.38
Q52	0~0
Q53	1.75~1.95
Q54	33.1~33.5

Q55	0~1
Q56	1~1.3
Q57	2500~2501
Q58	9.5~9.8
Q59	1325~1325
Q60	0.4~0.6

Hints & Solutions

Q1 Text Solution:

$$z=\frac{i-1}{\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}}\times\frac{\cos\frac{\pi}{3}-i\sin\frac{\pi}{3}}{\cos\frac{\pi}{3}-i\sin\frac{\pi}{3}}$$
$$=\frac{(i-1)\left(\cos\frac{\pi}{3}-i\sin\frac{\pi}{3}\right)}{\cos^2\frac{\pi}{3}+\sin^2\frac{\pi}{3}}$$
$$=\sqrt{2}\left(\cos\frac{5\pi}{12}+i\sin\frac{5\pi}{12}\right)$$

Correct Answer: B) $\sqrt{2}\left(\cos\frac{5\pi}{12}+i\sin\frac{5\pi}{12}\right)$

Q2 Text Solution:

Explanation:

Molar mass of K-40 Cl,

M=0.075 kg mol$^{-1}=75\ g\ mol^{-1}$

Number of molecules present in 1.2 mg of potassium chloride.

$N=\frac{m}{M}\times$Avogadro's number

$=\frac{1.2\times10^{-3}\times6.0\times10^{23}}{75}=9.6\times10^{18}$

Given $R=170s^{-1}$

But $R=\lambda N=\frac{0.693}{T_{1/2}}.N$

$\therefore\ T_{1/2}=\frac{0.693N}{R}=\frac{0.693\times9.6\times10^{18}}{170}$

$=3.91\times10^{16}s.$

Q3 Text Solution:

Concept:

The frequency of oscillation about a stable equilibrium point is

$\omega=\sqrt{\frac{k}{m}}$

where $k=\left.\frac{d^2V}{dx^2}\right|_{x=x_0}$, k is force constant.

Explanation:

Given-$V(x)=-ax+bx^2$

At equilibrium point, $\left.\frac{dV}{dx}\right|_{x=x_0}=0$

So, $-a+2bx_0=0$

$\Rightarrow x_0=\frac{a}{2b}$

Thus, $x_0=\frac{a}{2b}$ is an equilibrium point.

$\left.\frac{d^2V}{dx^2}\right|_{x=x_0}=2b>0$

So, $x_0=\frac{a}{2b}$ is stable equilibrium point.

Force constant $k=\left.\frac{d^2V}{dx^2}\right|_{x=x_0}=2b$

Therefore, frequency of oscillation is

$\omega=\sqrt{\frac{k}{m}}=\sqrt{\frac{2b}{m}}$

Hence, the correct answer is Option-A.

Q4 Text Solution:

The average energy is

$$<E>=\frac{-\varepsilon e^{-\varepsilon/k_BR}+0e^{-0/k_BT})+-\varepsilon e^{-\varepsilon/k_BR}}{e^{+\varepsilon/k_BT}+e^{-0/k_BT})+e^{-\varepsilon/k_BT}}$$

$$\frac{\varepsilon\left(e^{-\varepsilon/k_BT}-e^{-\varepsilon/k_BT}\right)}{e^{-\varepsilon/k_BT}+1+e^{\varepsilon/k_BT}}=\frac{\varepsilon\left(1-e^{2\varepsilon/k_BT}\right)}{1+e^{\varepsilon/k_BT}+e^{2\varepsilon/k_BT}}$$

$For\ T>>\frac{\varepsilon}{k_B}\ or\ \varepsilon<<k_BT,\ we\ have$

$$<E>=\frac{\epsilon[1-(1+2\epsilon/k_BT)]}{1+1+\epsilon/k_BT+1+2\epsilon/k_BT}=\frac{-2\epsilon^2/k_BT}{3+3\epsilon/k_BT}$$

$$\Rightarrow\left\langle E\right\rangle=-\frac{2\epsilon^2}{3k_BT}$$

$$\left[\therefore 1+\frac{\epsilon}{k_BT}\approx1\text{ for }\frac{\epsilon}{k_BT}\ll1\right]$$

Q5 Text Solution:

Concept:

- Resolving power of a diffraction grating:
  $R=\frac{\lambda}{\Delta\lambda}=mN$

where:

- m = diffraction order
- N = number of illuminated slits
- Minimum slits required to resolve two lines:
  $N=\frac{\lambda}{m\,\Delta\lambda}$

Solution:

1. Given: $\lambda = 500\,\text{nm} = 5 \times 10^{-7}\,\text{m}$,
$\Delta\lambda = 0.01\,\text{nm} = 1 \times 10^{-11}\,\text{m}$,
$m = 3$
2. Calculate N:
$N = \frac{\lambda}{m\,\Delta\lambda} = \frac{5\times10^{-7}}{3\cdot1\times10^{-11}} = \frac{5\times10^{-7}}{3\times10^{-11}} \approx 16,$
667

Q6 Text Solution:

Concept & Solution:

Use the FCC basis
$(0,0,0), (0,\frac{1}{2},\frac{1}{2}), (\frac{1}{2},0,\frac{1}{2}), (\frac{1}{2},\frac{1}{2},0)$.
The structure factor is proportional to
$S \propto 1 + (-1)^{k+l} + (-1)^{h+l} + (-1)^{h+k}$.
This sum equals 4 when h, k, l are either all even or all odd (so $S \neq 0$), and equals 0 for mixed parity (one or two odd, rest even). Thus reflections occur **only** when all three indices are all even or all odd.

Q7 Text Solution:

Concept:

- Low/Moderate Temperature: Electrons from donors dominate → $n \approx N_D, p \approx n_i^2/n$
- High Temperature (Intrinsic Region): Thermal generation dominates → $n \approx p \approx n_i(T)$
- Implication: Semiconductor behaves as intrinsic, independent of dopant type.

  - A → False, electron concentration is no longer ≈ N_D at high T
  - B → False, hole concentration increases with temperature
  - D → False, Fermi level moves toward intrinsic level, not donor level

Q8 Text Solution:

Concept & Reasoning:

Galilean transformation:
$x' = x - vt, \qquad t' = t$

Velocity transformation:
$u' = \frac{dx'}{dt} = \frac{d}{dt}\left(x - vt\right) = u - v$
Acceleration transformation:
$a' = \frac{du'}{dt} = \frac{du}{dt} = a$
So accelerations remain invariant in Galilean relativity.
Here,
$x\left(t\right) = 5t^2 \Rightarrow u\left(t\right) = 10t,$
$a\left(t\right) = 10\,\text{m/s}^2$
Thus in $S', a'\left(t\right) = 10\,\text{m/s}^2$

- A is false — acceleration does not change
- C is false — a shift in velocity does not change acceleration
- D is false — velocities differ by constant v

Hence B is the only correct statement.

Q9 Text Solution:

Solution:

Write the problem with borrowing:

$$\begin{array}{ccccccccc} & 1 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ - & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 \\ \hline \end{array}$$

Step-by-step subtraction (right → left):

Column	Operation	Result	Borrow
1 – 1	0	0	0
0 – 1	need borrow	1	borrow from next
0 – 1 (after borrow)	again borrow	1	borrow
1 – 0 (after double borrow)	1	0	
0 – 1	borrow	1	borrow
0 – 1 (after borrow)	borrow	1	borrow
0 – 0	0	0	
1 –1	0	0	

Final result: A: 00011010

Q10 Text Solution:

Concept:

The equation $z^3+1=0$ has three complex solutions $1,\ \omega\ and\ \omega^2$

Explanation:

The three roots satisfy $1+\omega+\omega^2=0$

where,

$\omega=\frac{-1+\sqrt{(-3)}}{2}$

and $\omega^2=\frac{-1-\sqrt{(-3)}}{2}$

Calculation:

Here $\frac{-1+\sqrt{(-3)}}{2}=\omega$ and $\frac{-1-\sqrt{(-3)}}{2}=\omega^2$

$\therefore \omega^{100}+\left(\omega^2\right)^{100}=\omega^{99}\omega+\omega^{198}\cdot\omega^2$

$=\omega+\omega^2=-1$

Hence, option (C) is correct.

Q11 Text Solution:

Solution:

Output voltage due to v_a: $v_{out1}=\frac{-250}{R_1}v_a$

output voltage due to v_b:

$v_{out2}=\left(1+\frac{250}{R_1}\right)\left(\frac{R_2}{R_1+R_2}\right)v_b$

so, $v_{out}=v_{out1}+v_{out2}=\frac{-250}{R_1}v_a$

$+\left(1+\frac{250}{R_1}\right)\left(\frac{R_2}{R_1+R_2}\right)v_b$

conpare with the given output:

$v_0=-5v_a+3v_b$

$\frac{-250}{R_1}=-5\Rightarrow R_1=50\ k\Omega$

$\left(1+\frac{250}{R_1}\right)\left(\frac{R_2}{R_1+R_2}\right)=3$

put value of R_1 *and solve*

$R_2=50\ k\Omega$

Q12 Text Solution:

Concept.

A photon with energy E scatters off a free electron at rest. After the collision the electron has kinetic energy K. From energy conservation the scattered photon energy is: $E'=E-K$ (since the electron final total energy is mc^2+K). The Compton relation between photon energies and the photon scattering angle φ is

$E' = \frac{E}{1+\frac{E}{mc^2}\left(1-\cos\varphi\right)}$.

Momentum conservation (vector) determines the electron recoil angle θ. Using components along and perpendicular to the incident photon direction gives: $p-p'\cos\varphi=p_e\cos\theta$,

$p'\sin\varphi=p_e\sin\theta$,

where $p=E/c,\ p'=E'/c,\ p_e$, is the electron momentum.

Solution:

Scattered photon energy:

$E'=E-K=1.250-0.400$

$=0.850\ \text{MeV}.$

Photon scattering angle φ:

$1-\cos\varphi=\left(\frac{E}{E'}\text{-}1\right)\frac{\text{mc}^2}{\text{E}}$

Putting all values we get:

$1-\cos\varphi=0.409\times0.471=0.193,$

$\varphi=\cos^{-1}\left(0.807\right)=36.136^\circ.$

Electron energy and momentum:

$E_e=mc^2+K=0.511+0.400$

$=0.911\ \text{MeV},$

$p_e=\sqrt{E_e^2-\left(mc^2\right)^2}=\sqrt{0.911^2-0.511^2}$

$=\sqrt{0.829-0.261}=\sqrt{0.569}$

$=0.754\ \text{MeV}/c.$

Electron recoil angle θ:

$\sin\varphi=\sin(36.136^\circ)=0.590,$

$\sin\theta=\frac{p'\sin\varphi}{p_e}=\frac{0.850\times0.590}{0.754}=0.665,$

$\theta=\sin^{-1}\left(0.665\right)=41.653^\circ.$

Final Result: $\varphi=36.136^\circ,\quad \theta=41.653^\circ$

Q13 Text Solution:

Solution Concept:

1. **Apogee radius:** $r_a=a(1+e)=1.8a$

2. **Circular orbit energy at apogee:**
$E_{\text{circ}} = -\frac{GMm}{2r_a} = -\frac{GMm}{3.6a}$

- Original elliptical energy: $E = -\frac{GMm}{2a}$
- Since $1/3.6a < 1/2a$, $|E_{\text{circ}}| < |E| \rightarrow$ **energy increases (less negative)**

3. **Angular momentum:**

- Circular:
$L_{\text{circ}} = m\sqrt{GMr_a} = m\sqrt{1.8GMa}$
- Elliptical:
$L = m\sqrt{GMa(1-e^2)} = m\sqrt{0.36GMa}$
- $1.8 > 0.36 \rightarrow$ **angular momentum increases**

Answer: (B)

Q14 Text Solution:

Concept:

For a small displacement $\mathbf{x}$ of the mass from the centre, the first-order change in length of a spring directed along unit vector $\hat{u}$ is:
$\Delta\ell \approx -\hat{u}\cdot\mathbf{x}.$

The spring gives a restoring **vector** force:
$\mathbf{F}_{\text{spring}} \approx -k\left(\hat{u}\cdot\mathbf{x}\right)\hat{u} = -k\left(\hat{u}\hat{u}^T\right)\mathbf{x}.$

Summing the forces from all springs gives $\mathbf{F} = -\mathbf{K}\mathbf{x}$ with stiffness matrix $\mathbf{K} = k\sum_i \hat{u}_i\hat{u}_i^T$. Diagonalize $\mathbf{K}$ (or use symmetry) to get the effective scalar stiffness k_{eff}. Then the small-oscillation equation is $m\ddot{\mathbf{x}} = -\mathbf{K}\mathbf{x}$ so ω^2 are the eigenvalues of $\mathbf{K}/m$.

Solution:

Geometry / unit vectors.

Place the square centre at the origin and the four vertices at $(\pm a, \pm a)$. The unit vectors from the centre to the vertices are (each of length $a\sqrt{2}$, so normalized):

$$\hat{u}_1 = \frac{1}{\sqrt{2}}\left(1,1\right),\quad \hat{u}_2 = \frac{1}{\sqrt{2}}\left(-1,1\right),$$
$$\hat{u}_3 = \frac{1}{\sqrt{2}}\left(-1,-1\right),\quad \hat{u}_4 = \frac{1}{\sqrt{2}}\left(1,-1\right).$$

Linearized extension.

For a small displacement $\mathbf{x} = (x, y)$, the extension of spring i is: $\Delta\ell_i \approx -\hat{u}_i\cdot\mathbf{x}$.

Potential energy (quadratic form).

Total spring potential (to second order) is:

$$V = \frac{1}{2}k\sum_{i=1}^4\left(\Delta\ell_i\right)^2 = \frac{1}{2}k\sum_{i=1}^4\left(\hat{u}_i\cdot\mathbf{x}\right)^2$$
$$= \frac{1}{2}\mathbf{x}^T\left(k\sum_{i=1}^4\hat{u}_i\hat{u}_i^T\right)\mathbf{x}.$$

So the stiffness matrix is: $\mathbf{K} = k\sum_{i=1}^4\hat{u}_i\hat{u}_i^T$.

Compute the dyadic sum explicitly.

Write a generic dyad for $\hat{u} = (\alpha, \beta)$:

$$\hat{u}\hat{u}^T = \begin{pmatrix}\alpha^2 & \alpha\beta\\ \alpha\beta & \beta^2\end{pmatrix}$$

Each $\hat{u}_i$ has $\alpha^2 = \beta^2 = \frac{1}{2}$ and $\alpha\beta = \pm\frac{1}{2}$ with signs that cancel pairwise. Summing the four dyads:

- Sum of xx-components: $4\times\frac{1}{2} = 2$
- Sum of yy-components: $4\times\frac{1}{2} = 2$
- Sum of off-diagonals: $\frac{1}{2}-\frac{1}{2}-\frac{1}{2}+\frac{1}{2} = 0$

Hence, $\sum_{i=1}^4\hat{u}_i\hat{u}_i^T = \begin{pmatrix}2 & 0\\ 0 & 2\end{pmatrix} = 2\,\mathbf{I}.$

$\mathbf{K} = k\cdot 2\,\mathbf{I} = 2k\,\mathbf{I}.$

Equation of motion and frequency.

Newton/Lagrange gives:
$m\ddot{\mathbf{x}} = -\mathbf{K}\mathbf{x} = -2k\,\mathbf{x}.$

Thus both components satisfy $m\ddot{x} = -2kx$, giving; $\omega^2 = \frac{2k}{m}$, $\quad\omega = \sqrt{\frac{2k}{m}}$.

Q15 Text Solution:

Since ab and cd both are adiabatic process, we have

$$p_iV_i^{\gamma} = p_1V_1^{\gamma}\quad\rightarrow(1)$$
$$p_fV_f^{\gamma} = p_2V_2^{\gamma}\quad\rightarrow\left(2\right)$$

and bc is isothermal process, so we have

$$P_1V_1 = P_2V_2 \Rightarrow \frac{P_1}{P_2} = \frac{V_2}{V_1}\quad\rightarrow\left(3\right)$$

Use equation (1) and (2)

$\frac{p_f V_f^{\gamma}}{p_i V_i^{\gamma}} = \frac{P_2 V_2^{\gamma}}{P_1 V_1^{\gamma}} = \left(\frac{V_2}{V_1}\right)^{\gamma-1}$

Given, $p_f = B^2 p_i$ and $V_f = \frac{V_i}{B}$

$\frac{B^2 p_f V_f^{\gamma}}{B^2 p_i V_i^{\gamma}} = \left(\frac{V_2}{V_1}\right)^{\gamma-1}$

$B^{2-\gamma} = \left(\frac{V_2}{V_1}\right)^{\gamma-1} \Rightarrow \frac{V_2}{V_1} = B^{\frac{2-\gamma}{\gamma-1}}$

For monoatomic gas $\gamma = \frac{5}{3}$

$\frac{V_2}{V_1} = B^{1/2}$

(a) $W_{ab} = \frac{1}{\gamma-1}\left[p_i V_i - P_1 V_1\right]$

$= \frac{1}{\gamma-1}\left[p_i V_i - RT_0\right] = \frac{3}{2}\left[p_i V_i - RT_0\right]$

(b) $W_{bc} = RT_0 \ \ln\left(\frac{V_2}{V_1}\right) = \frac{RT_0}{2} \ \ln \ B$

(c) $W_{cd} = \frac{1}{\gamma-1}\left[P_2 V_2 - p_f V_f\right]$

$= \frac{3}{2}\left[RT_0 - B^2 p_i \frac{V_i}{B}\right] = \frac{3}{2}\left[RT_0 - Bp_i V_i\right]$

The total work done by gas is,

$W = W_{ab} + W_{bc} + W_{cd}$

$W = \frac{3}{2}\left[p_i V_i - RT_0\right] + \frac{RT_0}{2} \ \ln \ B$

$+ \frac{3}{2}\left[RT_0 - Bp_i V_i\right]$

$W = \frac{RT_0}{2} \ \ln \ B + \frac{3}{2}(1 - B)p_i V_i$

Q16 Text Solution:

Zener voltage $V_z = 20V = V_L$.

maximum value of zener current,

$I_{zmax} = 50mA$

Load resistance, $R_L = 2k\Omega = 2000\Omega$

Load current, $I_L = \frac{V_L}{R_L} = \frac{V_z}{R_L} = \frac{20}{2000} = 10mA$

for zener to remain in on condition the voltage across it should remain $\geq 20V$. Assume minimum zener current is 0 mA.

$I_{s\,min} = I_{z\,min} + I_L = 0 + 10 = 10mA$

So the minimum input voltage,

$V_{s\,min} = I_{s\,min} \times R_s + V_z = 10 \times 10^{-3} \times 250 + 20 = 22.5$

$and \ I_{s\,max} = I_{z\,max} + I_L = 50 + 10 = 60 \ mA$

$V_{s\,max} = I_{s\,max} \times R_s + V_z = 35V$

Answer: 22.5V to 35V

Q17 Text Solution:

Concept:

The determinant is a special number that can be calculated from a matrix. It has important properties in the field of linear algebra.

Explanation:

Let's analyze each statement:

A. $det(AB) = det(A)det(B)$:

This statement is **true**. The determinant of the product of two matrices is equal to the product of their determinants.

B. $det(kA) = k^n det(A)$:

This statement is **true**. The determinant of a matrix multiplied by a scalar ‘k’ is equal to k raised to the power of ‘n’ (where ‘n’ is the order of the square matrix) times the determinant of the matrix.

C. $det(A + B) = det(A) + det(B)$:

This statement is **false**. The determinant of the sum of two matrices is not equal to the sum of their determinants. This is a common misconception.

D. $det\left(A^T\right) = \frac{1}{det\left(A^{-1}\right)}$:

This statement is **true**. The determinant of the transpose of a matrix ‘A’ is equal to the reciprocal of the determinant of the inverse of ‘A’, provided A is invertible.

Calculation:

For Statement C, consider a counterexample:

Let

$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \ and \ B = \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix}$

$det(A) = -2 \ and \ det(B) = -2$

$det\left(A + B\right) = det\left(\begin{bmatrix} 6 & 8 \\ 10 & 12 \end{bmatrix}\right) = -8$

$det(A) + det(B) = -2 + (-2) = -4$

as $det(A + B) \neq det(A) + det(B)$, option C is not true.

Q19 Text Solution:

Concept:

Since the two event happening at same point so,

$x_2 - x_1 = 0$

Formula for Lorentz transformation is

$t = \frac{t' + \frac{v}{c^2}x}{\sqrt{1 - \frac{v^2}{c^2}}}$

$x = \frac{x' + vt}{\sqrt{1 - \frac{v^2}{c^2}}}$

Explantion:

We have given the time difference in two event and happening at the same point in space we us the time difference formula for calculating the velocity and putting that in the Lorentz transformation of space we get the difference in space between two points.

Calculation:

$t_2 - t_1 = 6 \times 10^{-6}\ sec.$;

$t'_2 - t'_1 = 9 \times 10^{-6}\ sec.$

so

$\sqrt{1 - \frac{v^2}{c^2}}\left(t'_2 + \frac{v}{c^2}x_2 - \left(t'_1 + \frac{v}{c^2}x_1\right)\right) = 6 \times 10^{-6}$

$\sqrt{1 - \frac{v^2}{c^2}} = \frac{6 \times 10^{-6}}{t'_2 - t'_1} = \frac{6}{9}$

$\sqrt{1 - \frac{v^2}{c^2}} = \frac{6}{9} \Rightarrow v = \frac{\sqrt{5}}{3}c$

$x_2 - x_1 = \left(\frac{x'_1 + vt'_1}{\sqrt{1 - \frac{v^2}{c^2}}}\right) - \left(\frac{x'_2 + vt'_2}{\sqrt{1 - \frac{v^2}{c^2}}}\right)$

$x'_2 - x'_1 = \frac{v(t_2 - t_2)}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{\sqrt{5}}{3}c \times 6 \times 10^{-6} \times 1$

$.5 = 0.75 \times 3 \times 10^8 \times 9 \times 10^{-6}$

$x_2 - x_1 = 20.25\ \times 10^2 m \approx 2000\ m$

Q20 Text Solution:

Concept

A matrix M is **Hermitian** if $M^\dagger = M$ (conjugate transpose equals itself).

MM is **unitary** if $M^\dagger M = I$ (equivalently $M^{-1} = M^\dagger$).

If a matrix is **both**, its eigenvalues must be real (Hermitian) and have modulus 1 (unitary) — hence the only possibilities are eigenvalues ± 1. Also such matrices satisfy $M^2 = I$.

Solution

(A) $A = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$.

$A^\dagger = A$ (real symmetric) ⇒ Hermitian.

$A^2 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}^2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I$. Since $A^{-1} = A$ and $A^\dagger = A$, we have $A^\dagger A = I \Rightarrow$ unitary.

So **A is both Hermitian and unitary**.

(B) $B = \begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix}$.

$B^\dagger = \begin{pmatrix} 0 & -i \\ -i & 0 \end{pmatrix} \neq B \Rightarrow$ **not Hermitian**.

(Hence cannot be both.)

(C) $C = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$

$C^\dagger = C \Rightarrow$ Hermitian.

But $C^2 = \begin{pmatrix} 2 & 2 \\ 2 & 2 \end{pmatrix} \neq I$, and $C^\dagger C \neq I \Rightarrow$ **not unitary**.

(D) $D = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$.

$D^\dagger = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \neq D \Rightarrow$ **not Hermitian** (it is skew-symmetric). So not both.

Answer: **(A)** $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

Q21 Text Solution:

Concept:

Use Maxwell's equations in Gaussian units (vacuum, no sources):

- Faraday: $\nabla \times \mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t}$.

- Maxwell–Ampère: $\nabla \times \mathbf{B} = \dfrac{1}{c}\dfrac{\partial \mathbf{E}}{\partial t}$.

  Compute the relevant component that gives $\partial_t E_y$ and integrate in time.

Solution:

$\partial_t \mathbf{B} = -\omega B_0 e^{ax}\cos(ky-\omega t)\,\hat{z}$.

Faraday gives

$(\nabla \times \mathbf{E})_z = -\dfrac{1}{c}\partial_t B_z = \dfrac{\omega}{c} B_0 e^{ax}\cos.$

$(ky-\omega t)$

Compute $\nabla \times \mathbf{B}$. Since

$B_z = B_0 e^{ax}\sin(ky-\omega t)$.

$(\nabla \times \mathbf{B})_y = -\partial_x B_z = -aB_0 e^{ax}\sin$

$(ky-\omega t)$.

Maxwell–Ampère: $\dfrac{1}{c}\partial_t E_y = (\nabla \times \mathbf{B})_y$ so

$\frac{1}{c}\partial_t E_y = -aB_0 e^{ax}\sin\left(ky-\omega t\right)$.

Integrate in time:

$\partial_t E_y = -acB_0 e^{ax}\sin\left(ky-\omega t\right) \quad \Rightarrow$

$E_y = -\frac{ac}{\omega}B_0 e^{ax}\cos\left(ky-\omega t\right) + \text{const}.$

(Drop an arbitrary static constant if looking for the oscillatory solution.)

Answer: Option **(B)**:

$$\boxed{E_y = -\frac{ac}{\omega}\,B_0\,e^{ax}\cos\left(ky-\omega t\right).}$$

Q22 Text Solution:

Explanation:

In a system of non-interacting Fermi particles, the density of states (DOS) is given by:

$D(E) = \frac{dN}{dE} \propto \sqrt{E}$

The Fermi-Dirac distribution function f(E) describes the probability of a state with energy E being occupied by a fermion at temperature T:

$f(E) = \frac{1}{exp\left(\frac{E-\mu}{KT}\right)+1}$

Where:

μ is the chemical potential

K is the Boltzmann constant

T is the temperature

At T = 0, the distribution function becomes a step function, where all states below the Fermi energy are occupied and all states above are unoccupied. The Fermi energy E_f is the highest energy state occupied at T = 0.

Given that the density of states is proportional to $\sqrt{E}$, the number of states up to energy E is given by:

$N\left(E\right) = \int_0^E D(E')\,dE' \propto \int_0^E \sqrt{E'}\,dE'$

After integrating above expression we got-

$N \propto \frac{2}{3}\,E^{\frac{3}{2}}$

and also in term of fermi energy:

$N\,(E_f) \propto \frac{2}{3}\,{E_f}^{\frac{3}{2}}$

The average energy per particle at T = 0 is given by:

$\overline{E} = \frac{1}{N}\int_0^E E \times D\,(E)dE$

$D\,(E) \propto \sqrt{E}\ and\ N = \frac{2}{3}\,{E_f}^{\frac{3}{2}}$

$so,$

$\overline{E} = \frac{1}{N}\int_0^E E\sqrt{E}dE$

$\overline{E} = \frac{1}{N}\int_0^{E_f} E^{\frac{3}{2}}dE$

$\overline{E} = \frac{1}{N}\left(\frac{2}{5}E^{\frac{5}{2}}\right)_0^{E_f}$

$\overline{E} = \frac{1}{N}\left(\frac{2}{5}{E_f}^{\frac{5}{2}}\right)$

$Now\ putting\ the\ value\ of\ N\ and\ we\ get$

$\overline{E} = \frac{3}{5}E_f$

Q23 Text Solution:

The correct answer is **(a):** The sharp peak results from a complex interplay of isotope fractionation during vaporization and the Clausius-Clapeyron relationship for each isotope.

Option (b): Critical points only occur at specific pressure and temperature conditions, not within a coexistence region.

Option (c): Eutectic points typically involve two different components, not isotopes of the same element.

Option (d): While different ΔH_{vap} values might lead to slightly shifted transitions, a single, sharp peak wouldn't occur.

Q24 Text Solution:

Concept:

1. **Electric potential due to a volume charge density:** $V\left(\mathbf{r}\right) = \frac{1}{4\pi\varepsilon_0} \int \frac{\rho(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|} d^3r'$
2. For **spherically symmetric charge distributions**, the potential at a distance rr from the center is:
   $V\left(r\right)$
   $= \frac{1}{4\pi\varepsilon_0}\left[\int_0^r \frac{1}{r} 4\pi r'^2 \rho\left(r'\right) dr' + \int_r^R \frac{1}{r'} 4\pi r'^2 \rho\left(r'\right) dr'\right]$
3. **At the center** ($r = 0$), only the **second integral contributes** (all shells outside the point contribute):
   $V\left(0\right) = \frac{1}{4\pi\varepsilon_0} \int_0^R \frac{1}{r'} 4\pi r'^2 \rho\left(r'\right) dr' = \frac{1}{\varepsilon_0} \int_0^R r'^2 \frac{\rho(r')}{r'} dr' = \frac{1}{\varepsilon_0} \int_0^R r' \rho\left(r'\right) dr'$

Solution:

Given: $\rho(r') = kr'$

$V_0 = \frac{1}{\varepsilon_0} \int_0^R r' \cdot kr' dr' = \frac{k}{\varepsilon_0} \int_0^R r'^2 dr'$

$\int_0^R r'^2 dr' = \frac{R^3}{3}$

$\therefore V_0 = \frac{k}{\varepsilon_0} \cdot \frac{R^3}{3} = \frac{kR^3}{3\varepsilon_0}$

Q25 Text Solution:

Concept and solution:

To solve the problem of where the small black dot at the center of a solid glass sphere appears when viewed from outside, we can follow these steps:

1. **Understanding the Setup**:
   - We have a solid glass sphere with a refractive index μ.
   - A small black dot is located at the center C of the sphere.
2. **Light Rays Emission**:
   - When light rays emanate from the black dot at the center, they radiate outward in all directions.
3. **Refraction at the Surface**:
   - As these light rays reach the surface of the glass sphere, they will undergo refraction. However, since the dot is at the center, the rays that are emitted towards the surface are perpendicular to the surface at the point of incidence.
4. **Normal to the Surface**:
   - At the point where the rays hit the surface, the normal (a line perpendicular to the surface) is aligned with the direction of the rays coming from the center. Therefore, the angle of incidence is 0∘.
5. **Applying Snell's Law**:
   - According to Snell's law, $n_1 \sin\left(\theta_1\right) = n_2 \sin\left(\theta_2\right)$.
   - Here, n_1 is the refractive index of air (approximately 1), and n_2 is the refractive index of glass (μ).
   - Since the angle of incidence $\theta_1 = 0^\circ$, the sine of 0° is 0. Thus, the light rays will pass straight through without any deviation.
6. **Conclusion**:
   - Since the rays pass straight through the surface without bending, the black dot will appear to be located at the center C of the sphere when viewed from outside.
   - Therefore, regardless of the refractive index μ, the dot will always appear at the center.

Final Answer:

The dot will appear to be located at the center of the sphere.

Q26 Text Solution:

Concept:

Lissajous patterns are produced on a CRO screen when sinusoidal voltages of different frequencies are applied to the horizontal (X-axis) and vertical (Y-axis) deflection plates.

Explanation:

The frequency ratio $f_x : f_y$ can be determined as:

$f_x : f_y = \frac{\text{Number of loops along the } Y\text{-axis}}{\text{Number of loops along the } X\text{-axis}}$

Calculation:

(A)-(P):

$\frac{f_x}{f_y} = \frac{2}{1}$

$f_y = \frac{100}{2} = 50 \text{ Hz}$

$(B) - (S):$

$\frac{f_x}{f_y} = \frac{2}{3}$

$f_y = 100 \times \frac{3}{2} = 150 \text{ Hz}$

$(C) - (U):$

$\frac{f_x}{f_y} = \frac{1}{3}$

$f_y = 3 \times 100 = 300 \text{ Hz}$

The correct option is C

Q27 Text Solution:

Concept:

A conductor moving with velocity $\mathbf{v}$ through a magnetic field $\mathbf{B}$ experiences a motional emf between two points separated by $\mathbf{l}$:

$\mathscr{E} = \int(\mathbf{v} \times \mathbf{B}) \cdot d\mathbf{l}.$

Here $\mathbf{v}$ is along the track, $\mathbf{B}$ has a vertical component, and the rail separation L is horizontal and perpendicular to $\mathbf{v}$, so the emf between the rails simplifies to $\mathscr{E} = B\,v\,L$.

Solution:

$v = 90 \text{ km/hr} = \frac{90\times10^3}{3600} \text{ m/s} = 25 \text{ m/s}; B = 0.2 \text{ gauss} = 0.2 \times 10^{-4} \text{ T} = 2.0 \times 10^{-5} \text{ T}.$

Rail separation $L = 2 \text{ m}$.

Compute emf:

$\mathscr{E} = BvL = (2.0 \times 10^{-5} \text{ T})(25 \text{ m/s})(2 \text{ m}) = 1.0 \times 10^{-3} \text{ V} = 1.0 \text{ mV}.$

Correct option is **(B) 1**.

Q28 Text Solution:

Concept:

An ellipse is the locus of all those points in a plane such that the sum of their distances from two fixed points in the plane, is constant. The fixed points are known as the foci (singular focus), which are surrounded by the curve.

The shape of the ellipse is in an oval shape and the area of an ellipse is defined by its major axis and minor axis.

Area of ellipse=πab

where a and b are the length of semi-major and semi-minor axis of an ellipse.

The equation of the ellipse is given by

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

Explanation:

Given-$\begin{pmatrix} x & y \end{pmatrix} \begin{pmatrix} 5 & -7 \\ 7 & 3 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = 15$

$\Rightarrow 5x^2 + 3y^2 = 15$

$\Rightarrow \frac{x^2}{3} + \frac{y^2}{5} = 1$

Hence, the correct answer is Option-B.

Q29 Text Solution:

Concept:

The radius of gyration (k) is the distance from the axis of rotation at which the entire mass of an object can be considered to be concentrated to produce the same moment of inertia.

Explaination:

$k = \sqrt{\frac{I}{M}}$

I is the moment of inertia of the object about the given axis of rotation

M is the total mass of the object.

Calculation:

Moment of inertia of large sphere of mass M
$I_1 = \frac{2}{5} M a^2$
Moment of inertia of small sphere of mass m which is removed to hollow the large sphere
$I_2 = \frac{2}{5} m b^2$
radius of gyration
$k^2 = \frac{I}{(M-m)}$

The moment of inertia of the hollow sphere is derived by subtracting the moment of inertia of the smaller sphere (I_2) from the larger sphere (I_1)
$I = \frac{2}{5}\left(M a^2 - m b^2\right)$
volume density of large and small sphere
$M = \frac{4}{3}\pi \mathrm{a}^3 \rho$
$m = \frac{4}{3}\pi b^3 \rho$
$k = \sqrt{\frac{2}{5}\left(\frac{a^5 - b^5}{a^3 - b^3}\right)}$

Q30 Text Solution:

Concept:

-Three-dimensional cubic well: A confined space in the shape of a cube where a particle can exist.
-Impenetrable wallswalls: Boundaries that the particle cannot pass through.
-Energies of levels:The possible energy states the particle can occupy within the well.
-Degeneracy:The number of different quantum states that have the same energy level.

Explanation

The energy in three-dimensional cubic well can be written as
$E_n = \frac{\hbar^2 \pi^2}{2mL^2}\left[n_x^2 + n_y^2 + n_z^2\right]$
For third state $n_x = 2, n_y = 2$ and $n_z = 1$
$\Rightarrow E_3 = \frac{\hbar^2 \pi^2}{2mL^2}\left[2^2 + 2^2 + 1^2\right]$
$= \frac{9\pi^2 \hbar^2}{2mL^2}$
For fourth level $n_x = 3, n_y = n_z = 1$
$\Rightarrow E_4 = \frac{\hbar^2 \pi^2}{2mL^2}\left[3^2 + 1^2 + 1^2\right] = \frac{11\hbar^2 \pi^2}{2mL^2}$
Now,
$E = E_3 + E_4 = \frac{\pi^2 \hbar^2}{2mL^2}[9 + 11]$
$= \frac{10\pi^2 \hbar^2}{mL^2}$
For fourth level the possible values of n_x, n_y and n_z are $(3,1,1), (1,3,1)$ and $(1,1,3)$.
Hence, degeneracy of this level is 3 .
Hence (A) is correct option

Q31 Text Solution:

Concept

For a 2π-periodic function the Fourier expansion is
$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left(a_n \cos nx + b_n \sin nx\right)$,
with
$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx\, dx,\ b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx\, dx.$
If f is even (here x^2 is even), then all $b_n = 0$ and only cosine terms plus the constant term appear.

solution

1. Parity ⇒ sine coefficients vanish
x^2 is even, $\sin(nx)$ is odd ⇒ integrand $x^2 \sin(nx)$ is odd over $[-\pi, \pi]$. Hence for every n,
$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} x^2 \sin(nx)\, dx = 0.$
So **C is true**.

2. Constant term $a_0/2$
$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} x^2\, dx = \frac{2}{\pi}\int_0^{\pi} x^2\, dx = \frac{2}{\pi}\cdot\frac{\pi^3}{3} = \frac{2\pi^2}{3}.$
Thus the constant term (which is $a_0/2$) is
$\frac{a_0}{2} = \frac{1}{2}\cdot\frac{2\pi^2}{3} = \frac{\pi^2}{3}.$
So **A is true**.

3. Cosine coefficients $a_n$an for $n \geq 1$
Compute
$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} x^2 \cos(nx)\, dx = \frac{2}{\pi}\int_0^{\pi} x^2 \cos(nx)\, dx.$
Evaluate $I_n = \int_0^{\pi} x^2 \cos(nx)\, dx$ by two integrations by parts.
One convenient route:

Integrate by parts twice (or use known integrals).

The result is

$I_n = \frac{2\pi(-1)^n}{n^2}$.

Hence

$a_n = \frac{2}{\pi} I_n = \frac{2}{\pi} \cdot \frac{2\pi(-1)^n}{n^2} = \frac{4(-1)^n}{n^2}$.

So **B is true**.

(You can check for $n = 1$:

$a_1 = 4(-1)/1^2 = -4$ — matches the familiar $-4\cos x$ term.)

4. Are only odd harmonics present?

The formula $a_n = \frac{4(-1)^n}{n^2}$ is nonzero for **every** integer $n \geq 1$ (both even and odd). Therefore, the series contains **all** cosine harmonics $\cos nx$, not only odd ones. So **D is false**.

5. Full Fourier series (for reference)

$x^2 = \frac{\pi^2}{3} + 4\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2} \cos nx$.

Final answer

Correct statements: **A, B, C**.

(D is false.)

Q32 Text Solution:

Solution:

$$x = \left[\overline{\overline{A} + \overline{B.C}}\right] \oplus \left[\overline{\overline{C + D}\,.\,C}\right]$$

$$= \left(\overline{A} + \overline{B} + \overline{C}\right) \oplus \left(\overline{C}\,.\,\overline{D}\,.\,C\right)$$

$$= \left(\overline{A} + \overline{B} + \overline{C}\right) \oplus 0$$

$$\left(\overline{C}\,.\,C = 0\right)$$

$$= \overline{\left(\overline{A} + \overline{B} + \overline{C}\right).\,1} + \left(\overline{\overline{A} + \overline{B} + \overline{C}}\right).\,0$$

$$\left(A \oplus B = A\,\overline{B} + \overline{A}\,B\right)$$

$$= \overline{A} + \overline{B} + \overline{C}$$

$$= \overline{A.\,B.\,C}$$

Q33 Text Solution:

Concept:

For a potential step with $E > V_0$:

Wave numbers:

$$k_1 = \sqrt{\frac{E}{(\hbar^2/2m)}}, \quad k_2 = \sqrt{\frac{E-V_0}{(\hbar^2/2m)}}$$

Coefficients:

$$R = \left(\frac{k_1-k_2}{k_1+k_2}\right)^2, \qquad T = 1 - R$$

These arise from continuity of the wave function and its derivative at the step.

Solution:

Compute wave numbers:

$$k_1 = \sqrt{\frac{9}{3.81}} = \sqrt{2.363} = 1.538\ \text{Å}^{-1},$$

$$k_2 = \sqrt{\frac{9-4}{3.81}} = \sqrt{1.312} = 1.146\ \text{Å}^{-1}$$

Now,

$$R = \left(\frac{1.538-1.146}{1.538+1.146}\right)^2 = \left(\frac{0.392}{2.684}\right)^2 = \left(0.146\right)^2$$

$$\approx 0.0213$$

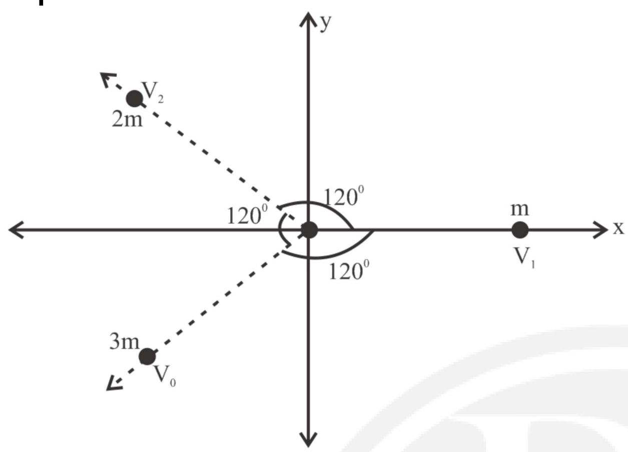
$$T = 1 - R = 1 - 0.0213 = 0.9787$$

Q34 Text Solution:

Concept:

The principle of momentum conservation states that the total momentum of an isolated system remains constant if no external forces act on it.

Explanation:



There is no external force acting on the object. Therefore, its momentum remains constant. Initial momentum is zero. Therefore, the vector sum of momenta of three fragments must be zero.

$\Rightarrow \vec{p} = 0$

$\Rightarrow p_x = 0,\ p_y = 0.$

Let the masses of three fragments be m, 2m and 3m.

Let v_1 and v_2 be the speed of fragments of masses m and 2m.

It is given that speed of 3m is v_0.

$\Rightarrow p_x = 0$

$\Rightarrow mv_1 - 2mv_2 \cos 60 - 3mv_0 \cos 60 = 0$

$\therefore v_1 - v_2 = \frac{3v_0}{2}$ ....(1)

$\Rightarrow p_y = 0$

$\Rightarrow 2mv_2 \sin 60 - 3mv_0 \sin 60 = 0$

$\therefore v_2 = \frac{3v_0}{2}$ ....(2)

from (1), we get

$\Rightarrow v_1 = \frac{3v_0}{2} + v_2$

$\Rightarrow v_1 = \frac{3v_0}{2} + \frac{3v_0}{2}$

$\Rightarrow v_1 = 3v_0$

Therefore, the speed of the lightest fragment is $3v_0$.

Q35 Text Solution:

Concept & solution:

First recall **Stokes' theorem** (vector form):

$\iint_S (\nabla \times \mathbf{F}) \cdot d\mathbf{S} \;=\; \oint_{\partial S} \mathbf{F} \cdot d\mathbf{r},$

where S is an oriented smooth surface and ∂S its positively oriented boundary.

Now treat each option quickly and precisely.

A. Reversing the orientation of ∂S changes the sign. — True.

Orientation of the boundary is tied to the surface orientation by the right-hand rule. Flipping the boundary orientation changes the sign of the line integral $\oint_{\partial S} \mathbf{F} \cdot d\mathbf{r}$, and so the two sides of Stokes' theorem pick up a minus sign. (check: $\oint_{-\partial S} = -\oint_{\partial S}$.)

B. If $\nabla \times \mathbf{F} = \mathbf{0}$, then the line integral around any closed loop is zero. — True.

Stokes' theorem gives

$\oint_{\partial S} \mathbf{F} \cdot d\mathbf{r} = \iint_S (\nabla \times \mathbf{F}) \cdot d\mathbf{S} = 0$ for any surface S whose boundary is the loop, provided $\nabla \times \mathbf{F} = 0$ everywhere on S. In practice this means the line integral is zero for any closed loop that bounds a surface lying entirely in the region where curl is zero (this is the usual simply-connected/domain condition used in potential theory).

C. If the line integral around a closed loop is zero, then $\nabla \times \mathbf{F} = \mathbf{0}$ everywhere. — False.

A single closed loop having zero circulation does **not** force the curl to vanish everywhere. Counterexample idea: take a field with nonzero curl at some point(s) but choose a loop that avoids those points — the loop integral can be zero while curl is nonzero elsewhere. (More sharply, even if all loops *in a region* have zero circulation, you can then infer zero curl in that region under suitable regularity and connectivity assumptions; but the statement as given — from one loop or without domain conditions — is false.)

D. Stokes' theorem is valid only for flat surfaces. — False.

Stokes' theorem holds for any sufficiently smooth, oriented surface (curved or flat); it is not restricted to planar surfaces. The theorem is precisely what lets you replace a surface integral over a curved surface with a line integral around its boundary.

Q36 Text Solution:

Concept:

For a **centered** charge, the potential it creates on the spherical surface is the **same constant** everywhere:

$V_q\left(R\right) = \frac{1}{4\pi\varepsilon_0}\frac{q}{R}$.

To make the conductor **grounded** ($V = 0$ on $r = R$), the induced charge on the inner surface simply supplies an **equal and opposite constant potential**. A uniform potential shift can be viewed in the method-of-images language as the effect of an **image charge at infinity**. No finite image point is required; the field inside remains purely radial as if only the real charge were present.

(For an **off-center** interior charge, you do need a finite image outside the sphere. The $q' = -\frac{R}{d}q,\ r' = \frac{R^2}{d}$ formula applies then, but it's undefined at $d = 0$, signaling the "image at infinity" case.)

Detailed solution

Let the real charge be at the origin; the shell is the sphere $r = R$ held at $V = 0$.

Assumption for the potential inside $r < R$.

By symmetry the potential depends only on r:

$V\left(r\right) = \frac{1}{4\pi\varepsilon_0}\frac{q}{r} + C,$

where C is a constant (comes from the surface charges on the conductor).

Apply boundary condition (grounded sphere).

Enforce $V(R) = 0$:

$0 = \frac{1}{4\pi\varepsilon_0}\frac{q}{R} + C \ \Rightarrow\ C = -\frac{1}{4\pi\varepsilon_0}\frac{q}{R}.$

Final interior potential.

$$V\left(r\right) = \frac{1}{4\pi\varepsilon_0}q\left(\frac{1}{r} - \frac{1}{R}\right), \qquad 0 < r < R.$$

Electric field inside ($r < R$).

$$\mathbf{E}\left(r\right) = -\nabla V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{\mathbf{r}}.$$

(Exactly the field of the lone central charge; the constant shift doesn't affect $\mathbf{E}$.)

Induced surface charge density.

The normal field just inside the surface is $E_r(R^-) = \frac{1}{4\pi\varepsilon_0}\frac{q}{R^2}$. For a grounded conductor,

$\sigma = -\varepsilon_0 E_r\left(R^-\right) = -\frac{q}{4\pi R^2},$

which is **uniform**. The total induced charge on the inner surface is $-q$ (any balancing charge can flow to ground, so the shell need not carry net charge).

Method-of-images interpretation

- For **interior, centered** charge: the only needed effect from images is a **constant potential offset → image at infinity** (choice C).
- For **interior, off-center** charge at distance d from center: a **single finite image outside** the sphere at $r' = R^2/d$ with $q' = -\frac{R}{d}q$ is required (not appliable at $d = 0$, hence the special "infinite" image here).

Answer is C.

Q37 Text Solution:

Concept:

Two or more different states of a system have the same energy level is called degenracy.

Calculataion:

Three mode of vibration so 3 degree of freedom

Energy of system is given by

$E=\left(n_1+\frac{1}{2}\right)\hbar\omega_1+\left(n_2+\frac{1}{2}\right)\hbar\omega_2$

$+\left(n_3+\frac{1}{2}\right)\hbar\omega_3$

$\omega_2=2\omega_1\ and\ \omega_3=3\omega_1$

$E=\left(n_1+2n_2+3n_3+\frac{1}{2}+\frac{2}{2}+\frac{3}{2}\right)\hbar\omega_1$

It is given that the enrgy is less than the $\frac{11}{2}\hbar\omega_1$

$E_0=\left(0+0+0+\frac{6}{2}\right)\hbar\omega_1$

$E_0=3\hbar\omega_1$

$E_1=\left(1+0+0+\frac{6}{2}\right)\hbar\omega_1$

$E_1=4\hbar\omega_1$

$E_2=\left(2+0+0+\frac{6}{2}\right)\hbar\omega_1$

$E_2=5\hbar\omega_1$

$E_2=\left(0+2\times1+0+\frac{6}{2}\right)\hbar\omega_1$

$E_2=5\hbar\omega_1$

Hence n_1=2 and n_2=2 having same energy so these are degenrate states.

The genrating function is given by

$Z=\sum_i g_i e^{-\beta E_i}$

$Z=e^{-\beta3\hbar\omega}+e^{-\beta4\hbar\omega}+2\times e^{-\beta5\hbar\omega}$

Q38 Text Solution:

Concept:-

Group velocity is the speed at which the envelope of a wave packet or the overall modulation of waves moves through space.

Explanation:-

For no dispersion,

$\mathrm{v}_g=\mathrm{v}$, i.e., $\frac{d\mathrm{v}}{d\lambda}=0$

Differentiating the expression $\mathrm{v}^2=\frac{g\lambda}{2\pi}+\frac{2\pi\sigma}{\rho\lambda}$, with respect to λ and setting $\frac{d\mathrm{v}}{d\lambda}=0$ and $\lambda=\lambda_0$

gives $\lambda_0=2\pi\sqrt{\sigma/\rho g}$

Calculation:-

Substituting the values of g,σ,ρ and λ_0 yields,

$\lambda_0=2\pi\sqrt{\sigma/\rho g}=2\pi\sqrt{\frac{7.2\times10^{-2}}{1000\times9.8}}=0.017$

$\lambda_0=1.7\times10^{-2}\ \mathrm{m}=1.7\ \mathrm{cm}$

Thus, waves of average wavelength of 1.7 cm do not disperse in water.

The group and phase velocities at λ_0 are equal to each other.

$\mathrm{v}=\mathrm{v}_g=\left(\frac{g\lambda_0}{2\pi}+\frac{2\pi\sigma}{\rho\lambda_0}\right)^{1/2}$

Substituting the values of g,σ,ρ and λ_0

$\mathrm{v}_g=\left(\frac{9.8\times0.017}{2\pi}+\frac{2\pi\times7.2\times10^{-2}}{1000\times0.017}\right)^{1/2}=0.23\ \mathrm{m}/\mathrm{s}$

$\mathrm{v}_g=23\mathrm{cms}^{-1}$

Q39 Text Solution:

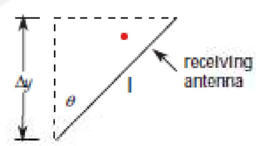
Concept:

The question explores how the orientation of a receiving dipole antenna affects the received power from a vertically oriented transmitting antenna. It involves the relationship between antenna alignment and power reception.

Explanation:

When the receiving dipole antenna is tilted from the vertical, its ability to capture power from the vertically oriented transmitting antenna decreases, as the received power depends on the relative alignment of the antennas. The power received varies with the cosine of the tilt angle.

Solution:



We konw that power is given by

$P=\frac{V}{R}^2$

$P\propto\Delta V^2$

$\Delta V=-E_y.\Delta y.$

$E_y l\cos\theta$

$P\propto\cos^2\theta$

at zero 30 degree

$=\cos^2(30)\ =\frac{3}{4}\times100$

=75%
similarly for
45 degrees = 50%
and for 90 degrees =0%

Q40 Text Solution:

Concept & Solution:

1. p-orbital details:

- $l=1 \Rightarrow m_l=-1,0,+1$ → three spatial orbitals.
- Each orbital can hold 2 electrons with opposite spins ($m_s=\pm\frac{1}{2}$) → p-subshell maximum 6 electrons.
  - B is correct.

2. Pauli exclusion principle:

- No two electrons in the same orbital can have identical n, l, m_l, m_s.
- So in the same orbital, at most one spin-up and one spin-down.
  - A is incorrect, since 3 electrons cannot all have $m_s=+1/2$ in the same orbital.
  - D is correct, at least two electrons in the same orbital must have opposite spins.

3. Total spin S:

- With three electrons in three orbitals, the spins can combine to form S = 3/2 (all parallel) or S = 1/2 (one anti-aligned).
  - C is correct.

Q41 Text Solution:

Concept:
Bragg diffraction occurs only when Bragg's Law is satisfied: nλ=2dsinθ
For a physically valid reflection: sinθ≤1
So the maximum order is found from:

$n_{max}\lambda \le 2d \quad \Rightarrow \quad n_{max}=\left\lfloor \frac{2d}{\lambda} \right\rfloor$
For cubic crystal:
Interplanar spacing for (hkl):
$d_{hkl}=\frac{a}{\sqrt{h^2+k^2+l^2}} \Rightarrow d_{111}=\frac{a}{\sqrt{3}}=\frac{5.64}{\sqrt{3}}\ \text{Å}$
Maximum order:
$n_{max}=\left\lfloor \frac{2d_{111}}{\lambda} \right\rfloor \Rightarrow \left\lfloor \frac{2\times3.256}{1.54} \right\rfloor=\left\lfloor \frac{6.512}{1.54} \right\rfloor$
$=\lfloor 4.23 \rfloor=4$

Q42 Text Solution:

To solve the circuit assuming ideal diodes, let's break down the problem step by step:
Circuit Overview:
- There are two voltage sources: 5V and 8V.
- The circuit has two diodes D_1 and D_2, each connected through 1kΩ resistors.
- We need to find the current through diode D_2.

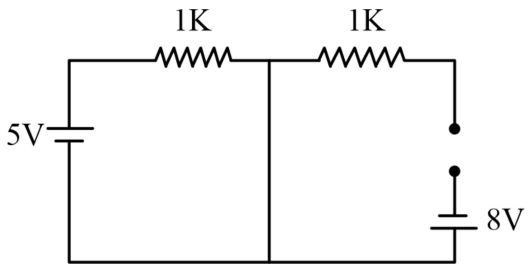
Step 1: Analyzing the Diode Status
- **Ideal diodes** conduct when they are forward biased (i.e., when the anode voltage is greater than the cathode voltage), and they act like short circuits in this case.
- When reverse biased, diodes act as open circuits.

For Diode D_1:
- The anode of D_1 is connected to the 5V source, and the cathode is connected to the 8V source.
- Since the cathode of D_1 is at a higher voltage (8V) than the anode (5V), D_1 is reverse biased and will not conduct. It behaves as an open circuit.

For Diode D_2:
- The anode of D_2 is connected to the 5V source, and the cathode is connected to ground (0V).
- Since the anode of D_2 is at a higher voltage (5V) than the cathode (0V), D_2 is forward biased and will conduct. It behaves like a short circuit.

Step 2: Calculate the Current Through D_2
- Since D_2 is forward biased and acts like a short circuit, the current through D_2 is determined by

the 5V supply and the 1kΩ resistor in series with it.



- Using Ohm's Law:

$$I_{D_2} = \frac{V_{source}}{R} = \frac{5V}{1k\Omega} = 5\,\text{mA}$$

Step 3: Conclusion
- The current through D_2 is **5 mA**.

Thus, the current in D_2 is 5 mA, while D_1 does not conduct any current since it is reverse biased.

Q43 Text Solution:

Concept:

The peak wavelength of blackbody radiation follows **Wien's displacement law**:

$\lambda_{\max} T = \text{constant.}$

$\frac{\lambda_1}{\lambda_2} = \frac{T_2}{T_1}.$

Given data:

- $T_1 = 6000\,\text{K}$
- $\lambda_1 = 600\,\text{nm} = 6 \times 10^{-7}\,\text{m}$
- $T_2 = 300\,\text{K}$
- Find λ_2.

Calculation:

$\lambda_2 = \lambda_1 \times \frac{T_1}{T_2} \Rightarrow \lambda_2 = \left(6 \times 10^{-7}\right) \times \frac{6000}{300}$

$\lambda_2 = 6 \times 10^{-7} \times 20 = 1.2 \times 10^{-5}\,\text{m}$

$1.2 \times 10^{-5}\,\text{m} = 12\,\mu\text{m}.$

Q44 Text Solution:

Concept:

The center of mass (COM) of a system is the point where the weighted relative position of the distributed mass sums to zero. For a linear distribution, such as students along a rope, the COM is found by multiplying each mass by its position and dividing by the total mass.

Explanation:

Assuming the students are evenly spaced along the rope, we can assign positions based on their order. Let's say they are positioned at 1 m intervals, starting from 0 m for the 60 kg student to 4 m for the 100 kg student.

Calculation:

The position of the COM (x_{COM}) is given by:

$x_{\text{COM}} = \frac{\sum(m_i \cdot x_i)}{\sum m_i}$

Where (m_i) is the mass of the $\left(i^{th}\right)$ student and (x_i) is the position of the $\left(i^{th}\right)$ student.

Let's calculate it:

$x_{\text{COM}} = \frac{60\cdot0+70\cdot1+80\cdot2+90\cdot3+100\cdot4}{60+70+80+90+100}$

$x_{\text{COM}} = \frac{0+70+160+270+400}{400}$

$x_{\text{COM}} = \frac{900}{400}$

$x_{\text{COM}} = 2.25\text{ m}$

Conclusion: The center of mass of the group lies **2.25 m** from the 60 kg student's position, assuming they are spaced 1 m apart along the rope.

Q45 Text Solution:

Given equation:

$2x^2 + 2y^2 + 8x - 20y + 10 = 0$

Divide the entire equation by 2:

$x^2 + y^2 + 4x - 10y + 5 = 0$

Complete the square for x and y:

$x^2 + 4x + 4 + y^2 - 10y + 25 = 4 + 25 - 5$

$(x+2)^2 + (y-5)^2 = 24$

Standard form of the circle's equation:

$(x-h)^2 + (y-k)^2 = r^2$

Comparing with the standard form:

$r^2 = 24$

$r = \sqrt{24} = 2\sqrt{6}$

The area of the square whose side equals the radius of the circle:

$A = side^2$

$A = (2\sqrt{6})^2$
$A = 4 \times 6$
$A = 24$
The area of the square is 24 square units.

Q46 Text Solution:

Concept and solution:

1. Dipole moment:
$p = 2aQ = (2 \times 0.10)(2.0 \times 10^{-6})$
$= 4.0 \times 10^{-7}$ C.m.
2. Position vector:
$x = 0.60,\ y = 0.80, \quad r = \sqrt{0.6^2 + 0.8^2}$
$= 1.0$ m.
3. Dipole potential formula: $V = \frac{k\vec{p}\cdot\hat{r}}{r^2} = \frac{k\,p\,x}{r^3}$.

Since $\vec{p} = p\hat{i}$ and $\vec{r} = x\hat{i} + y\hat{j}$

4. Substitution: $V = \frac{(9\times10^9)(4.0\times10^{-7})(0.60)}{(1.0)^3}$.

$V = 2.16 \times 10^3$ V.

Q47 Text Solution:

Concept & solution:

When the switch closes at $t = 0$:

- The capacitor initial voltage is $V(0) = V_0 = E_1 = 10.0$ V.
- The final steady voltage (as $t \to \infty$) across the capacitor is $V_\infty = E_2 = 2.0$ V because the capacitor behaves like an open circuit in steady DC and the node is held at the supply E_2.
- The time constant of the $R - C$ network is: $\tau = RC = (1.0 \times 10^3\ \Omega)$ $\cdot (1.0 \times 10^{-6}\ \text{F}) = 1.0 \times 10^{-3}$ s $= 1.0$ ms.
- The general transient for a capacitor charging/discharging toward a new final voltage is: $V(t) = V_\infty + (V_0 - V_\infty)\,e^{-t/\tau}$.

Plug numbers in:

- $V_0 - V_\infty = 10.0 - 2.0 = 8.0$ V.
- $t = 2.4$ ms $= 2.4 \times 10^{-3}$ s.
- $t/\tau = \frac{2.4\ \text{ms}}{1.0\ \text{ms}} = 2.4$.
- Add the final voltage:
$V(t) = V_\infty + 0.725 = 2.0 + 0.7257$
$= 2.725$ V.

V(2.4 ms)≈2.73 V.

Q48 Text Solution:

Concept:

The concept involves the ideal gas law and kinetic theory of gases. Using the gas law to find temperature and the relation $KE_{avg} = \frac{3}{2}k_BT$, where k_B is Boltzmann's constant, the average kinetic energy is calculated.

Explanation:

Using the ideal gas law $PV = nRT$, temperature (T) is determined. Then, the average kinetic energy of gas molecules is calculated using $KE_{avg} = \frac{3}{2}k_BT$, linking molecular motion to temperature.

Calculation:

Volume occupied by gas
$= 7 \times 10^{-3}\ m^3$
PV=nRT
$T = \frac{PV}{nR}$
$= \frac{1.6\times10^6\ \times7\times10^{-3}}{3.5\times8.31}$
$= 385\ K$

Average kinetic energy per molecule in the gas
$= \frac{3}{2}k_BT$
$= \frac{3}{2} \times 1.38 \times 10^{-23} \times 385$
$= 0.797 \times 10^{-20}\ J$

Q49 Text Solution:

Concept:

Energy is conserved for the block-spring system between the maximum-displacement and the half-maximum points .

So

$$(K.E+P.E)_{in}=(K.E+P.E)_f$$
$$0+\frac{1}{2}\times 6.5\times(0.1)^2=\frac{1}{2}\times m\times(0.3)^2$$
$$+\frac{1}{2}\times 6.5\times(0.05)^2$$
$$m=\frac{2\times 0.0244}{0.09}$$
$$m=0.542\ kg$$

Q50 Text Solution:

From S_1 frame the velocity of the particle is 0.8cj.

So energy is

$$E_1=\frac{m_0c^2}{\sqrt{1-\frac{v^2}{c^2}}}=\frac{m_0c^2}{\sqrt{1-0.64}}$$
$$E_1=1.66\ m_0c^2$$
$$u_x=\frac{u_x'+v}{1+\frac{u_x'v}{c^2}}=0.8c$$
$$u_y'=\frac{u_y\sqrt{1-\frac{v^2}{c^2}}}{1+\frac{u_x'v}{c^2}}=0.8c\sqrt{1-0.64}$$
$$u_y'=0.48c$$
$$u=\sqrt{(u_x')^2+(u_y')^2}$$
$$u=\sqrt{(0.8c)^2+(0.48c)^2}$$
$$u=0.93c$$
$$E_2=\frac{m_0c^2}{\sqrt{1-0.87}}=2.78\ m_0c^2$$
$$\frac{E_2}{E_1}=\frac{2.78}{1.67}$$
$$\frac{E_2}{E_1}=1.66$$

Q51 Text Solution:

Concept and Solution:

For a compensated semiconductor, the electron concentration n is approximately N_D-N_A. The Fermi level relative to the intrinsic level is:

$$E_F-E_i=kT\ln\frac{n}{n_i}=kT\ln\frac{N_D-N_A}{n_i}.$$

Substitute values:

$$n=N_D-N_A=5\times10^{16}-2\times10^{16}=3\times10^{16}\ \mathrm{cm}^{-3}$$
$$\frac{n}{n_i}=\frac{3\times10^{16}}{1.5\times10^{10}}=2\times10^6$$
$$\ln(2\times10^6)=\ln(10^6)+\ln 2\approx 13.816+0.693=14.509$$
$$E_F-E_i=0.0259\times14.509\approx0.375\ \mathrm{eV}$$

The closest option is 0.36 eV.

Q52 Text Solution:

$$\hat{p}=-i\hbar\frac{d}{dx}$$
$$\langle p\rangle=\int_{-\infty}^{\infty}\psi^*(x)\left(-i\hbar\frac{d}{dx}\right)\psi(x)dx$$

For the ground state, the Hermite polynomial $H_0(x)$ is equal to 1, so the wave function simplifies to:

$$\psi(x)=Ae^{-\alpha x^2}$$
$$\langle p\rangle=\int_{-\infty}^{\infty}A^2e^{-\alpha x^2}\left(-i\hbar\frac{d}{dx}\right)e^{-\alpha x^2}dx$$

Since the wave function is an even function, its derivative will be an odd function. The product of an even function and an odd function is an odd function, and the integral of an odd function over symmetric limits is zero. Therefore:

$$\langle p\rangle=\int_{-\infty}^{\infty}A^2e^{-\alpha x^2}\left(-i\hbar\frac{d}{dx}\right)e^{-\alpha x^2}dx=0$$

Q53 Text Solution:

Concept:

Continuity equation:

$$A_1v_1=A_2v_2\quad\Rightarrow\quad v_2=v_1\frac{A_1}{A_2}=v_1\frac{D_1^2}{D_2^2}$$

Bernoulli's theorem (horizontal pipe → neglect height difference):

$$P_1+\frac{1}{2}\rho v_1^2=P_2+\frac{1}{2}\rho v_2^2\quad\Rightarrow$$
$$P_2=P_1+\frac{1}{2}\rho(v_1^2-v_2^2)$$

Solution:

Velocity at narrow section:

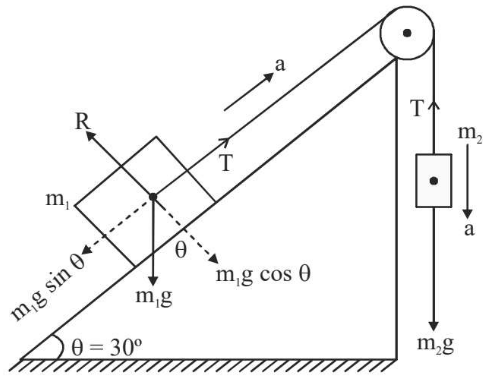
$$v_2=v_1\frac{D_1^2}{D_2^2}=1.5\frac{0.10^2}{0.05^2}=1.5\frac{0.01}{0.0025}=1.5\times4$$
$$=6.0\ \mathrm{m/s}$$

Pressure at narrow section:

$$P_2=P_1+\frac{1}{2}\rho(v_1^2-v_2^2)=2.0\times10^5+0.5\times1000\times(1.5^2-6.0^2)$$
$$P_2=2.0\times10^5-16875=183125\ \mathrm{Pa}$$
$$\approx1.83\times10^5\ \mathrm{Pa}$$

Q54 Text Solution:

The different forces acting on the masses are shown in figure.



We have T – m_1g sinθ = m_1a ...(i)

and m_2g – T = m_2a ...(ii)

Solving equations (i) and (ii), we get

$a = \frac{(m_2 - m_1 \sin\theta)g}{(m_1+m_2)}$

and T=$m_2g\left[1 - \frac{(m_2 - m_1 \sin\theta)}{(m_1 - m_2)}\right]$

or $T = \frac{m_1 m_2(1+\sin\theta)g}{(m_1+m_2)}$

Here m_1= 4 kg, m_2= 5 kg, θ = 30° and g = 10 m/s

Substituting these values in equations (iii), we get

$T = \frac{5xr\left(1+\frac{1}{2}\right)10}{9} = \frac{300}{9} = 33.33N$

Q55 Text Solution:

Concept:

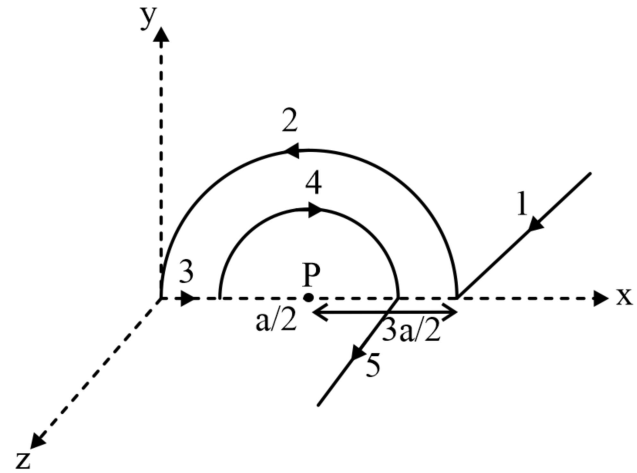
This problem combines concepts from electromagnetism, such as the Biot-Savart Law, the magnetic field due to circular arcs, and the superposition principle.

Explanation:

To calculate the magnetic field at point P due to each segment of the current-carrying wire in the diagram, we apply the Biot Savart's law that states that, the magnetic field $\overrightarrow{\mathrm{d}B}$ at a point in space due to an infinitesimal current element $I\overrightarrow{\mathrm{d}l}$ is given by,

$\overrightarrow{dB} = \frac{\mu_0}{4\pi}\frac{Id\vec{l}\times\hat{r}}{r^2}$

Calculation:



The net magnetic field at point P is,

$\vec{B}_P = \vec{B}_1 + \vec{B}_2 + \vec{B}_3 + \vec{B}_4 + \vec{B}_5$

$\vec{B}_1 = \frac{\mu_0 I}{4\pi\left(\frac{3a}{2}\right)}\left(-\hat{j}\right)$

$\Rightarrow \vec{B}_1 = \frac{\mu_0 I}{6\pi a}\left(-\hat{j}\right)$

$\vec{B}_2 = \frac{\mu_0 I}{4\left(\frac{3a}{2}\right)}\hat{k}$

$\Rightarrow \vec{B}_2 = \frac{\mu_0 I}{6a}\hat{k}$

$\vec{B}_3 = 0$

$\vec{B}_4 = \frac{\mu_0 I}{4\left(\frac{a}{2}\right)}\left(-\hat{k}\right)$

$\Rightarrow \vec{B}_4 = \frac{\mu_0 I}{2a}\left(-\hat{k}\right)$

$\vec{B}_5 = \frac{\mu_0 I}{4\pi\left(\frac{a}{2}\right)}\left(-\hat{j}\right)$

$\Rightarrow \vec{B}_5 = \frac{\mu_0 I}{2\pi a}\left(-\hat{j}\right)$

Now, Adding all of the magnetic fields at point P to get the net magnetic field at P,

$\vec{B} = -\frac{\mu_0 I}{6\pi a}\hat{j} + \frac{\mu_0 I}{6a}\hat{k} + 0 - \frac{\mu_0 I}{2a}\hat{k} - \frac{\mu_0 I}{2\pi a}\hat{j}$

$\Rightarrow \vec{B} = \frac{\mu_0 I}{2a}\left(-\frac{1}{3\pi} - \frac{1}{\pi}\right)\hat{j} + \frac{\mu_0 I}{2a}\left(\frac{1}{3} + 1\right)\hat{k}$

$\Rightarrow \vec{B} = \frac{\mu_0 I}{2a}\left(-\frac{4}{3\pi}\hat{j} + \frac{4}{3}\hat{k}\right)$

$\Rightarrow \vec{B} = \frac{2\mu_0 I}{3}\left(-\frac{1}{\pi}\hat{j} + \hat{k}\right)$

$\left|\vec{B}\right| = 0.6\frac{\mu_0 I}{a} \times \sqrt{\left(-\frac{1}{\pi}\right)^2 + 1^2}$

$\left|\vec{B}\right| = 0.6\frac{\mu_0 I}{\pi a}\sqrt{1+\pi^2}$

Thus, on compairing the value for x = 0.6

Q56 **Text Solution:**

The rms velocity is given by

$$V_{rms} = \sqrt{\frac{3RT}{m}}$$

$$V_{rms} \propto \sqrt{\frac{T}{m}}$$

In this case T (temperature) is also fixed,

$$V_{rms} \propto \sqrt{\frac{1}{m}};\ only\ depends\ on\ molar\ mass$$

$$\frac{(V_{rms})_{N_2}}{(V_{rms})_{O_2}} = \sqrt{\frac{m_{O_2}}{m_{N_2}}}$$

Put the values

$$\frac{(V_{rms})_{N_2}}{(V_{rms})_{O_2}} = \sqrt{\frac{32}{28}}$$

$$\frac{(V_{rms})_{N_2}}{(V_{rms})_{O_2}} = \sqrt{1.14}$$

$$\frac{(V_{rms})_{N_2}}{(V_{rms})_{O_2}} = 1.1$$

Q57 **Text Solution:**

Concept:

For a **non-relativistic Doppler shift** (when $v \ll c$):

$$\frac{\Delta\lambda}{\lambda_0} = \frac{v}{c}$$

where

λ_0 = rest wavelength,

$\Delta\lambda$ = observed shift in wavelength,

v = velocity of the star,

c = speed of light.

Solution:

Given: $\lambda_0 = 5000\ \text{Å}, \quad \Delta\lambda = 2\ \text{Å}$

Using the Doppler relation:

$$\frac{v}{c} = \frac{2}{5000} \Rightarrow \frac{c}{v} = \frac{5000}{2} = 2500$$

Q58 **Text Solution:**

Concept:

For a reaction: $A + B \longrightarrow C + \gamma$

Q-value: $Q = \left(m_A + m_B - m_C\right)c^2$

Total kinetic energy after reaction:

$K_{\text{final}} = K_{\text{initial}} + Q$

If photon recoil is neglected, all kinetic energy goes to the nucleus C: $K_C = K_{\text{initial}} + Q$

Solution:

Mass difference:

$\Delta m = (m_{\text{n}} + m_{\text{C12}}) - m_{\text{C13}}$

$= (1.008 + 12.000) - 13.003 = 13.008$

$- 13.003 = 0.005\ \text{u}$

Q-value:

$Q = \Delta m \times 931.494 = 0.005 \times 931.494$

$\approx 4.657\ \text{MeV}$

Final kinetic energy:

$K_C = K_{\text{initial}} + Q = 5.0 + 4.657$

$= 9.657\ \text{MeV}$

$\boxed{K_C \approx 9.7\ \text{MeV}}$

Q59 **Text Solution:**

Concept:

With the emitter effectively at ground (leakage negligible) the base-emitter junction drops V_{BEQ}. The resistor R_B must drop the remaining voltage $V_{BB} - V_{BEQ}$ while supplying the desired base current I_B. Ohm's law gives:

$R_B = \frac{V_{BB} - V_{BEQ}}{I_B}$.

Solution:

Voltage across R_B:

$V_{RB} = V_{BB} - V_{BEQ} = 6.0 - 0.7 = 5.3\ \text{V}$.

Required resistor:

$R_B = \frac{5.3\ \text{V}}{4\times10^{-6}\ \text{A}} = 1.325 \times 10^6\ \Omega$.

In kilo-ohms: $R_B = 1325\ \text{k}\Omega = 1.325\ \text{M}\Omega$.

Answer: $R_B \approx 1325\ \text{k}\Omega$

Q60 **Text Solution:**

Concept:

The jacobian is

$$\frac{\partial(u,v)}{\partial(x,y)} = \left\|\begin{matrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} \end{matrix}\right\|$$

Explanation:

Let $u = x^2 - y^2,\ v = x^2 + y^2$

$1 \le u \le 2,\ 2 \le v \le 4$

$\frac{\partial(u,v)}{\partial(x,y)} = \left\| \begin{matrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} \end{matrix} \right\| = \left\| \begin{matrix} 2x & -2y \\ 2x & 2y \end{matrix} \right\|$

$= |4xy + 4xy| = 8xy$

$du.\,dv = \frac{\partial(u,v)}{\partial(x,y)} dx.\,dy$

$du.\,dv = 8xydx.\,dy$

$dx.\,dy = \frac{du.dv}{8xy}$

$u = x^2 - y^2,\ v = x^2 + y^2$

$u + v = 2x^2,\ v - u = 2y^2$

$x^2 = \frac{u+v}{2},\ y^2 = \frac{v-u}{2}$

Calculation:

$I = \iint_R x^3y^3dx.\,dy = \int_{v=2}^{4}\int_{u=1}^{2} x^3y^3\frac{du.dv}{8xy}$

$I = \int_{v=2}^{4}\int_{u=1}^{2} \frac{1}{8}x^2y^2du.\,dv$

$I = \int_{v=2}^{4}\int_{u=1}^{2} \frac{1}{8} \times \frac{1}{2}\left(u + v\right)$

$\times \frac{1}{2}\left(v - u\right)du.\,dv$

$I = \frac{1}{32}\int_{v=2}^{4}\int_{u=1}^{2}\left(v^2 - u^2\right)du.\,dv$

$I = \frac{1}{32}\int_{v=2}^{4}\left[v^2u - \frac{u^3}{3}\right]_{u=1}^{2} dv$

$I = \frac{1}{32}\int_{v=2}^{4}\left(2v^2 - \frac{8}{3}\right) - \left(v^2 - \frac{1}{3}\right)dv$

$I = \frac{1}{32}\int_{v=2}^{4}\left(v^2 - \frac{7}{3}\right)dv$

$I = \frac{1}{32}\left[\frac{v^3}{3} - \frac{7}{3}v\right]_2^4$

$I = \frac{1}{32}\left[\left(\frac{64}{3} - \frac{28}{3}\right) - \left(\frac{8}{3} - \frac{14}{3}\right)\right]$

$I = \frac{1}{32} \times \frac{42}{3} = \frac{7}{16}$

$I = 0.4375$